JUNIOR HIGH SCHOOL MATHEMATICS
BOOK II

WENTWORTH-SMITH MATHEMATICAL SERIES

JUNIOR HIGH SCHOOL MATHEMATICS

BOOK II

BY

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PREFACE

A proper curriculum for junior high schools and six-year high schools demands, in the opinion of many teachers, a course in mathematics which presents concrete, intuitional geometry and the simple uses of algebra in the introductory stages. This book is intended to meet this demand for the second year of the introductory course.

Algebra furnishes the material for the first half of the book, the second half being devoted to those topics of business arithmetic which are appropriate to the student's maturity. The work in Book I has already shown the use of the formula in algebra and has furnished a large amount of material to which the formula can now properly be still further related. It is therefore natural to place algebra in the first half year and to make free use of it in the arithmetic that follows, unless the courses are carried along parallel to each other.

The work in algebra is such as every boy, and every girl as well, should become familiar with at this time. It represents that which each will meet in ordinary reading, and although it is not burdened with the technical phraseology of the shop it is utilitarian in the best sense. The formula is needed in reading books and articles of various kinds, the graph is used in many lines of business and study, the equation is helpful in manipulating the formula, and the negative number is so commonly used as to be part of the necessary equipment of every reader of current literature as well as of scientific books. These are, therefore, the features upon which the greatest emphasis is placed in the work of the first half year.

Any remodeling of the elementary curriculum that sacrificed thorough training in arithmetic would be a transitory thing,

and any anæmic course in mathematics that leaves the student too languid intellectually to pursue the subject further with success is foredoomed to failure. This book gives to arithmetic the place due to it because of its fundamental importance; it adheres to a sane and usable topical plan throughout the development of the various subjects treated; and because of this the authors believe that they have here produced a textbook suited to the needs of a rapidly growing class of schools and feel confident that they have not failed in any respect to adhere to the best standards of teaching.

In this book it will be seen that both the algebra and the arithmetic make use of the important facts presented in Book I, and that each includes those large and important topics which are valuable in the elementary education of every boy and girl. The two books thus work together to a common purpose, the first being more concrete and preparing by careful steps for the second, and the second blending with the first in presenting a well-organized foundation for the more formal mathematics which naturally follows. The two form such an introduction to mathematics as will enable a decision to be reached as to whether or not the student is fitted to continue with profit his work in this important intellectual field; they open the door to mathematics, showing its general nature and its purpose, and they thus present that preliminary and general view which it should be the privilege of every student to have and which it is the duty of every school to give at some time in the course. Such a general view of the purposes of mathematics is especially valuable for students whose work terminates at the close of this second year and who are thereby deprived of the advantages of a further high-school course.

Teachers are advised not to require all the exercises, to notice carefully the work on pages 239–244, and to select and vary the requirement from time to time.

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JUNIOR HIGH SCHOOL MATHEMATICS

BOOK II

PART I. ALGEBRA

I. THE FORMULA

Abbreviations and Symbols. In all your mathematics you have been using a kind of shorthand. You have used the symbol + instead of writing the word "plus," the symbol ÷ for "divided by," and the symbol $\sqrt{\ }$ for "the square root of." Moreover, instead of writing "the area of a rectangle is equal to the product of the base and height," you have learned to write simply

$$A = bh.$$

All this is a kind of shorthand and is designed to save time. We could get along without it, just as we could get along without the trolley car and the telephone, but we should lose time by so doing. Algebra saves time in learning and writing the necessary rules of number needed in the shop, the bank, the insurance office, and many other places.

We use many other symbols for numbers, as *doz.* (dozen) for 12, *cwt.* (hundredweight) for 100 lb., and M for 1000.

Students who have not studied Book I should be told how to evaluate an expression like bh when $b = 12$ and $h = 5$, and should be led to understand the meaning of the formula $A = bh$.

Exercise 1. Abbreviations and Symbols

All work oral

1. How many oranges are $6\frac{1}{2}$ doz. oranges?

2. How many dozen eggs are $2\frac{1}{2}$ doz. eggs and $3\frac{1}{2}$ doz. eggs? How many eggs?

3. How many envelopes are 7 C envelopes and 9 C envelopes?

Envelopes are commonly counted by the hundred (C) or thousand (M), and the symbols C and M are used in marking the boxes.

4. A bond dealer sends a bill to a customer for 5 M bonds of one kind and 8 M of another kind. How many thousand dollars worth of bonds were purchased?

It is the custom to write bills in this way, 5 M or 5 ~~M~~, meaning 5 bonds of $1000 each.

5. How many reams of paper are 10 R and 20 R, where R stands for reams? If R = 500, how many sheets of paper are there in all?

A ream of folded note paper is often taken as 20 quires (20 qr.) or 480 sheets, but in unfolded sheets a ream is usually taken as 500 sheets. Paper in bulk is usually sold by the pound.

6. How many pounds of paper are 80 lb. and 50 lb.?

State the amount of paper in each of the following cases:

7.	**8.**	**9.**	**10.**
3 qr.	3 lb.	3 R	3 M sheets
7 qr.	7 lb.	7 R	7 M sheets
10 qr.	10 lb.	10 R	10 M sheets

In Ex. 7 state the result both in quires and in sheets. In Ex. 9 take R to mean reams of 500 sheets, and state the result both in reams and in sheets. In Ex. 10 take M to mean 1000, as is usual in all commercial work.

Simplifying. Since algebra is in part a kind of shorthand, it is desirable to shorten or *simplify* all expressions whenever we can. The following are examples:

2 doz. $+ 3$ doz. is simplified by writing 5 doz.,
$2x + 7x + 9x$ is simplified by writing $18x$,
$a + a + a + a$ is simplified by writing $4a$,

and $aaaa$, which means $a \times a \times a \times a$, is simplified by writing a^4, which is read, "a to the fourth power."

The expressions "coefficient" and "exponent" may be informally introduced if desired, but they are not needed at present. The use of a dot as a symbol of multiplication, as in $a \cdot b$, or $3 \cdot 4 \cdot 5$, may be mentioned, although it is not needed in our present work.

Exercise 2. Simplifying

All work oral

1. A grocer bought 7 doz. $+ 4$ doz. $+ 9$ doz. eggs. Simplify this statement, stating the number of dozen eggs.

2. In Ex. 1 how many eggs did the grocer buy?

3. Simplify $7x+4x+9x$. State its value when $x=12$.

4. Simplify $2a+5a+3a$. State its value when $a=10$.

5. A stationer who had a box containing $5\,\mathrm{C}$ envelopes sold $2\,\mathrm{C}$ envelopes. How many envelopes had he left?

6. Simplify $5\,\mathrm{C} - 2\,\mathrm{C}$; $5x - 2x$; $5m - 2m$; $12a - 7a$.

7. Simplify $15\,\mathrm{M} - 10\,\mathrm{M} - 2\,\mathrm{M}$ and state the value of the expression when $\mathrm{M} = 1000$.

8. A square is 3 in. on a side; how many square inches in the area? A square is s inches on a side; how many square inches in the area?

9. Simplify $a \times a$; $x \times x$; $y \times y$; 2×2; $2a \times 2a$.

The expression $a \times a = a^2$, read, "a square"; $2a \times 2a = 4a^2$.

Evaluating. It is convenient to have a single word to use for the expression "Find the value of," and we use the word *evaluate* for this purpose.

For example, to evaluate bh for the values $b = 6$ and $h = 4$ we have $bh = 6 \times 4 = 24$. That is, $bh = 24$ for these values of b and h.

Similarly, we evaluate a^2, which means aa, for $a = 7$ by writing 7 for a, whence $a^2 = 7 \times 7 = 49$.

Exercise 3. Evaluating

Examples 1 to 3, oral

1. The area of a square of side s is s^2; evaluate this expression for $s = 11$.

The 11 may stand for inches, feet, yards, or any other unit of length. In algebra we do not ordinarily state the unit.

2. Evaluate bh for $b = 8$, $h = 4$.

3. Evaluate $\frac{1}{2}bh$ for $b = 10$, $h = 6$.

4. A playground is l feet long and w feet wide. What is the area of the playground in square feet? Evaluate the answer for $l = 150$ and $w = 75$.

Students who have not studied Book I may have the formula given them. It is reviewed later.

In this book, playgrounds, rooms, boxes, fields, and the like are to be taken as rectangular unless the contrary is expressly stated.

Evaluate the following for $a = 7$, $b = 8$:

5. $3a + b$. **7.** $a^2 + b$. **9.** $a^2 + b^2$. **11.** ab.

6. $5b + 9$. **8.** $b^2 + a$. **10.** $3a^2 + 1$. **12.** $3a^2b$.

Evaluate the following for $a = 3$, $b = 2\frac{1}{2}$, $x = 9$:

13. a^4. **15.** $x \div a$. **17.** abx. **19.** x^2.

14. $6b$. **16.** $3a \div x$. **18.** $bx \div a$. **20.** a^2x.

Further Uses of Symbols. We shall now translate a few sentences into symbols and a few symbols into sentences.

1. Express in symbols the statement that a certain number is to be increased by 5, and the sum is to be multiplied by 7.

Letting n stand for the number, we have

$$7(n+5).$$

This means that the operation $n+5$, in the parentheses, is first to be performed and that the sum is then to be multiplied by 7.

2. Express in symbols the statement that the square of a certain number is to be increased by 3, and the sum is to be divided by 2.

We may write $\frac{x^2+3}{2}$, $(x^2+3)\div 2$, or $(x^2+3)/2$.

The fraction form shows that x^2+3 is all to be divided by 2, but in the other cases we have to use parentheses because $x^2+3\div 2$ means that only the 3 is to be divided by 2.

3. Translate into words the expression $\frac{ab}{4}+7$.

This means that the product of two numbers which we do not know is to be divided by 4, and that 7 is to be added to the quotient.

4. Express in symbols the statement that 7 is to be diminished by some number which we do not know and the result multiplied by itself.

We have $(7-x)(7-x)$ or $(7-x)^2$.

5. Express in symbols the statement that some number which we do not know is to be divided by 9 and that 5 is to be added to the quotient.

We have $\frac{x}{9}+5$.

6. Find the value of the result in Ex. 5 if $x=27$.

We have $\frac{27}{9}+5=3+5=8$.

Exercise 4. Statements and Symbols

Express each of the following statements in symbols:

1. Some number is increased by 7.

2. Some number is diminished by 9.

3. To some number there is added 6 and the sum is divided by 5.

4. From some number there is taken 4, and the difference is multiplied by $2\frac{1}{2}$.

5. Some number is multiplied by $2\frac{3}{4}$, the product is divided by 3, and this quotient is then increased by 9.

Express in words the meaning of each of the following:

6. $\dfrac{3x}{4}$.

7. $2x+1$.

8. $3\frac{1}{2}x-7$.

9. $\dfrac{4+x}{3}$.

10. $5-\frac{1}{2}x$.

11. $4+\frac{3}{4}x^2$.

12. $\dfrac{x-3}{x+1}$.

13. $0.7x+2$.

14. $2.8x-7$.

15. If n is a whole number, what is the whole number next larger than n? the whole number next smaller than n? the three numbers that are twice as large as n and twice as large as each of the other two numbers?

16. If n is a whole number, is $2n$ an even number or an odd number? How do you know? Consider the same questions with relation to $2n+1$ and $2n-1$.

17. Write in algebraic form the quotient of any odd number divided by 5.

18. From a steel beam weighing 850 lb. there hangs a block and tackle weighing 225 lb., and by this a load of L pounds is being lifted. What is the total resulting load on the beam, including the weight of the beam itself?

Need for Formulas. As we found in Book I, a formula like $A = bh$ is merely a kind of shorthand. This shorthand is so convenient that it is used in shops and in various kinds of commercial and industrial work and is needed in reading trade journals, books on mechanics, and various articles in encyclopedias.

For example, a tinsmith knows that the rule for finding the number of cubic inches in a cylindric cup is as follows: The number of cubic inches is equal to $\frac{1}{4}$ of $\frac{22}{7}$ times the number of inches of height multiplied by the square of the diameter. To learn such a rule, however, or even to read it, is a waste of time when the whole thing is given in this formula:

$$V = \tfrac{1}{4}\pi h d^2,$$

where V is the volume in cubic inches, h the number of inches of height, d the number of inches of diameter, and π (Greek letter pī) approximately $\frac{22}{7}$, or 3.1416.

It is not necessary at this time to explain how the rule or formula is obtained. It is desirable merely that the student should see the value of algebraic shorthand.

To take another example: A manufacturer has found that the number of workmen he needs to employ to make a certain number of machines in a certain number of days is expressed by the simple formula

$$w = \frac{6\,m}{d},$$

where w is the number of workmen, m the number of machines, and d the number of days. If he wishes to turn out 25 machines in 5 da., he must employ $\dfrac{6 \times 25}{5}$ workmen, or 30 workmen.

The student should state the formula as a rule and thus see its advantage over the rule.

Exercise 5. Formulas

Examples 1 to 3, oral

1. Translate into words this formula for the area of a rectangle when the length and width are given: $A = lw$.

2. Translate into words this formula for the width of a rectangle when the area and length are given: $w = A \div l$.

We may write $A \div l$ in other ways, such as $\frac{A}{l}$ or A/l.

3. If 1 yd. of cloth costs 20¢, how much do 7 yd. cost? If 1 yd. of cloth costs c cents, how much do n yards cost? State a formula for C, the cost.

It is sometimes, as here, convenient to use both C and c in the same formula. In such a case we may speak of large c and small c. Sometimes we distinguish between two uses of the same letter by writing c and c' ("c-prime").

In such examples the prices are supposed to vary at the same rate.

4. Write a rule for the area of a square when the side is known, and write the formula.

5. Write a rule for the area of the entire surface of a cube when the edge is known, and write the formula.

6. Write a rule for finding the number of inches in a given number of feet, and write the formula.

Let i represent the number of inches required and f the given number of feet. Check by showing that the rule is true when $f = 1$.

7. The number of times that a bicycle wheel of known circumference in feet will turn in going a mile is given by the formula $N = 5280/c$. Translate this into words.

8. The formula $d = 1.23\sqrt{h}$ is used in determining the distance in miles that can be seen at sea from an elevation of h feet. Find this distance for elevations of 9 ft., 16 ft., 25 ft., and 100 ft.

Formulas in Games. Boys are naturally familiar with formulas used in baseball. If we let P stand for the number of put outs, A for the number of assists, E for the number of errors, and G for the number of games, we have

$$\text{Total fielding chances} = P + A + E.$$

That is, if the Detroit team had 4062 put outs in a season, 1966 assists, and 212 errors, the total fielding chances were

$$P + A + E = 4062 + 1966 + 212 = 6240.$$

If they played 153 games the fielding chances per game were

$$\frac{P + A + E}{G} = \frac{6240}{153} = 40.78.$$

The accepted chances were $P + A$, and hence the "fielding percentage" $= \dfrac{P + A}{P + A + E} = \dfrac{6028}{6240} = 0.966.$

That is, the per cent of accepted chances to total chances was 96.6. The put outs are usually represented by PO, but in an algebraic formula we need but one letter for this purpose.

This page may be used only with the boys if desired.

Exercise 6. Formulas in Games

Find the fielding percentages in the following cases:

1. St. Louis, $P = 4191$, $A = 2065$, $E = 308$.
2. Cleveland, $P = 4109$, $A = 2042$, $E = 280$.
3. Washington, $P = 4183$, $A = 1911$, $E = 230$.

Find the fielding chances per game in the following cases:

4. New York, $P = 4135$, $A = 2008$, $E = 217$, $G = 154$.
5. Detroit, $P = 4230$, $A = 2170$, $E = 258$, $G = 156$.

Formulas learned in Geometry. In the geometry which was studied in Book I many formulas were found. Some of these will now be reviewed.

Area of a Rectangle. As already stated, the number of units of area of a rectangle is equal to the product of the number of units of length and the number of units of width, or of the base and height. We may therefore express this in the shorthand of algebra as a formula, thus:

$$A = lw,$$

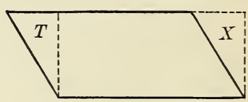
or

$$A = bh.$$

The student should translate each formula into words.

It is not the custom in mathematics to use the expression "number of units" each time. These words are understood, and we shall generally omit them.

Area of a Parallelogram. We learned in geometry that the area of a parallelogram is equal to the product of the base and height. Hence we may express this in algebraic form thus:

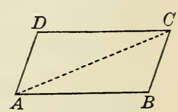


$$A = bh.$$

For the triangle T may be cut off and placed at X, the parallelogram then becoming a rectangle.

If $b = 5$ and $h = 3\frac{1}{2}$, then $a = bh = 5 \times 3\frac{1}{2} = 17\frac{1}{2}$; that is, we *substitute* the values of b and h in the formula. If b and h represent inches, then a represents square inches.

Area of a Triangle. We have learned in geometry that the area of a triangle is equal to half the product of the base and height. This may be expressed in algebraic form thus:



$$A = \frac{1}{2} bh.$$

For the triangle in this figure is seen to be half of the parallelogram of base b and height h.

If $b = 7$ and $h = 10\frac{1}{2}$, we substitute these values for b and h in the formula, and $a = \frac{1}{2} bh = \frac{1}{2}$ of $7 \times 10\frac{1}{2} = 36\frac{3}{4}$.

Exercise 7. Rectangle, Parallelogram, Triangle

1. A playground is l feet long and w feet wide. Find the area in square feet. What is the number of square feet when $l = 160$ and $w = 75$?

2. A triangular gable at the end of a house, under the roof, is f feet wide and h feet high. What is the area in square feet? What is the area when $f = 20$ and $h = 6\frac{1}{2}$?

3. A corner lot is in the form of a parallelogram of base n feet and height m feet. Find the area. What is the area when $n = 96$ and $m = 48\frac{1}{2}$?

4. Translate into words the formula $A = \frac{1}{2}bh$, referring to the area of a triangle.

5. To find the number of square inches in the area of a rectangle f feet long and i inches wide, take the product of 12 times the number of feet in the length by the number of inches in the width. Express this as a formula.

6. If A is the number of square feet of area of a rectangle, and the dimensions in inches are b and h, then $A = \frac{1}{144}bh$. Express this formula in words as a rule.

Given $A = bh$, find in each case the value of A when:

7. $b = 6.4$, $h = 2.4$.

8. $b = 7.8$, $h = 3.5$.

9. $b = 228\frac{1}{2}$, $h = 38\frac{3}{4}$.

10. $b = 42.75$, $h = 24.8$.

Given $A = \frac{1}{2}bh$, find in each case the value of A when:

11. $b = 64$, $h = 36$.

12. $b = 48$, $h = 18$.

13. $b = 84$, $h = 32$.

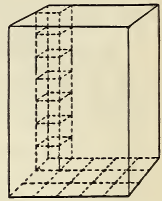
14. $b = 46.5$, $h = 8.7$.

15. $b = 62.2$, $h = 38.5$.

16. $b = 48.3$, $h = 34.7$.

17. Translate into words the formula $h = 2A \div b$, which refers to the height of a triangle.

Volume of a Rectangular Solid. If this figure represents a rectangular solid 5 in. long, 3 in. wide, and 7 in. high, it is evident that in the column of cubes shown there are 7 cu. in. It is also evident that on the base we can place 3×5 such columns. Hence the volume is $3 \times 5 \times 7$ cu. in., or 105 cu. in. Therefore,



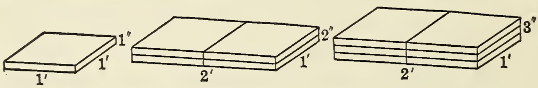
The volume of a rectangular solid is equal to the product of the three dimensions.

This may be expressed by the formula

$$V = lwh.$$

We read 2 ft. × 3 ft. × 4 ft. "two feet by three feet by four feet," and this is a proper method of expressing dimensions.

Measuring Lumber. A board foot (bd. ft.) of lumber means a piece that is 1 sq. ft. on one surface and 1 in. or less in thickness. A board foot is often called a foot.



1 BOARD FOOT 4 BOARD FEET 6 BOARD FEET

A board 24′ long, 10″ wide, and 1″ or less thick contains $24 \times \frac{10}{12}$ bd. ft., or 20 bd. ft. A beam 16′ long, 6″ wide, and 8″ thick contains $16 \times \frac{6}{12} \times 8$ bd. ft., or 64 bd. ft. A board 12 ft. long, 8 in. wide, and $\frac{7}{8}$ in. thick contains $12 \times \frac{2}{3}$ bd. ft., or 8 bd. ft. A fraction of a board foot is counted as 1 bd. ft.

We may express the number of board feet as follows:

$$B = \frac{lwt}{12},$$

where B is the number of board feet, l the length in feet, w the width in inches, and t the thickness in inches.

Exercise 8. Rectangular Solid

1. A box is $4\frac{1}{2}$ in. long, $3\frac{1}{2}$ in. wide, and $1\frac{1}{4}$ in. deep. Write the formula for the volume and find the volume.

In the case of boxes and other receptacles, all dimensions represent inside measures except when the contrary is stated. In each problem of this kind the result should be estimated in advance as a check on the accuracy of the computation.

2. An excavation is 48 ft. long, 36 ft. wide, and 6 ft. 4 in. deep. Write a formula for the number of loads (cubic yards) of earth removed and then find this number.

3. A pile of 4-foot wood is 64 ft. long and 6 ft. high. Write a formula for the number of cords (128 cu. ft.) in the pile and then find this number.

4. A coal bin 18 ft. long and 8 ft. wide is filled with coal to the depth of 6 ft. Allowing 35 cu. ft. of coal to a ton, write a formula for the number of tons of coal in the bin, and then find this number.

5. A boy measured the planks in the floor of a barn near his home. Each plank was 12′ long, 1″ wide, and 2″ thick, and 136 planks were used. By substituting in the formula find how many feet of lumber were used.

6. A boy found that a sill was 16′ long, 8″ wide, and 10″ thick. By substituting in the formula find how many feet of lumber the sill contained.

By substituting in the formula find the number of feet of lumber in each of the following:

7. 8 boards, each 16′ long, 10″ wide, and 1″ thick.

8. 32 planks, each 14′ long, 1′ wide, and 2″ thick.

9. 36 boards, each 16′ long, 8″ wide, and $\frac{7}{8}$″ thick.

10. 48 joists, each 14′ long, 8″ wide, and 3″ thick.

Exercise 9. Review

1. If one bicycle costs d dollars, how much will n bicycles cost at the same rate? Evaluate the formula for $d = 18$, $n = 12$; for $d = 20$, $n = 0$.

2. At the rate of m miles an hour, how many miles will an automobile go in h hours? Evaluate the formula for $m = 23$, $h = 2\frac{1}{2}$.

3. At the rate of m miles an hour, how long will it take a company of Boy Scouts to go to a place 3 mi. away and return to the starting place? Evaluate the formula for $m = 3$; for $m = 2\frac{3}{4}$.

4. At the rate of m miles an hour, how long will it take a man to walk to a place d miles away and return to the starting place? Evaluate the formula for $m = 3$, $d = 7$; for $m = 3\frac{1}{2}$, $d = 10\frac{1}{2}$.

5. If n tennis rackets cost d dollars, what is the average cost? Evaluate the formula for $n = 18$, $d = 45$.

The *average* of n numbers is found by dividing their sum by n, just as the average cost of 7 objects is found by dividing their total cost by 7.

6. If one hat costs $3, how much will n such hats cost?

7. If n hats of the same kind cost d dollars, how much will one such hat cost?

8. If n hats of the same kind cost d dollars, how much will 5 such hats cost?

9. If n hats of the same kind cost d dollars, how much will a such hats cost? Write the formula as a rule.

10. In order to save d dollars in n years, how much must a man's savings average per month? Write the formula as a rule. Evaluate the formula for $d = 960$, $n = 2$.

11. A playground is l feet long and w feet wide. Find the area of the playground in square feet and also in square yards. Then copy and complete the following table:

Length, l feet	Width, w feet	Formula		Area	
		Sq. ft.	Sq. yd.	Sq. ft.	Sq. yd.
120	60	lw	$\frac{1}{9}lw$	7200	800
160	40				
175	65				

12. A lot containing A square feet is l feet long. Find the width. As in Ex. 11, make out a table giving length, width, formula, and area for $A = 3000$, $l = 75$; for $A = 4500$, $l = 90$; for $A = 9600$, $l = 120$.

13. Find the length of fence required to inclose the lot mentioned in Ex. 12, and make out a table for the cases there given.

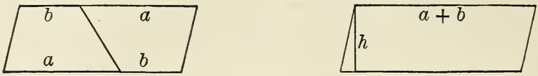
14. In heating iron red hot, it expands $\frac{3}{16}$ in. per foot. If a casting is f feet long before it is heated, how long is it when made red hot? Make out a table for the cases $f = 1\frac{1}{2}$; $f = 1\frac{3}{4}$; $f = 2\frac{1}{2}$; $f = 3\frac{1}{4}$.

15. If a train travels m miles in h hours, what is the average rate per hour? Make out a table for the cases $m = 30$, $h = \frac{3}{4}$; $m = 75$, $h = 2\frac{1}{2}$; $m = 135$, $h = 3\frac{3}{4}$.

16. If your marks in algebra are m on Monday, t on Tuesday, and w on Wednesday, what is your average mark for the 3 da.? Make out a table for the cases $m = 100$, $t = 90$, $w = 90$; $m = 100$, $t = 80$, $w = 95$.

17. If an automobile travels m miles per hour, what is the rate in feet per hour? in feet per minute? in feet per second? Find the results when $m = 20$; when $m = 24$.

Area of a Trapezoid. It often happens that it is necessary to find the area of a piece of land in the form of a trapezoid. You learned in Book I how to find the area of a trapezoid, and we shall now review the subject.



If we place two equal trapezoids end to end, one being turned over, as in the figure at the left, they together form a parallelogram of twice the area of either trapezoid. The base is then the sum of the upper and lower bases of the trapezoid, and the height is the height of the trapezoid.

Therefore if the two bases of the trapezoid are a and b and the height is h, the area of the parallelogram is the product of h and $a+b$. Hence the area of the trapezoid is half the area of the parallelogram. That is,

$$A = \tfrac{1}{2}\,h\,(a+b),$$

where the parentheses show that the operation $a+b$ is to be performed first.

That is, if $h = 8$, $a = 10$, and $b = 7$, we have

$$A = \tfrac{1}{2} \times 8 \times (10 + 7) = 4 \times 17 = 68.$$

Order of Operations. Always remember that the *operations within the parentheses are to be performed first.* After that the following is the order of operations:

1. *Powers and roots.*

2. *Multiplications and divisions in the order in which they occur.*

3. *Additions and subtractions in the order in which they occur.*

Thus $2 + 3^2 \times 8 \div 2 \times 5 = 2 + 72 \div 2 \times 5 = 2 + 36 \times 5 = 2 + 180 = 182.$

Exercise 10. Area of a Trapezoid

1. A piece of land is in the form of a trapezoid with bases 180 ft. and 160 ft. and with height 80 ft. Write the formula and find the area of the piece of land.

2. A piece of land is in the form of a trapezoid with bases $2b$ and b and with height h. Find the area. Evaluate the result for $b = 100$, $h = 90$; $b = 150$, $h = 125$; $b = 200$, $h = 175$; $b = 150$, $h = 150$.

3. A flower bed in a park is in the form of a trapezoid with bases $3b$ and b and with height $2b$. Find the area. Evaluate the result for $b = 25$.

4. A playground is in the form of a trapezoid with bases a and b and with height c. Find the area. Evaluate the result for $a = 150$ ft., $b = 125$ ft., and $c = 92$ ft.

5. In Ex. 4 evaluate the result for $a = 60$ yd., $b = 40$ yd., and $c = 35$ yd.; for $a = 50$ yd., $b = 30$ yd., and $c = 45$ yd.

Evaluate the formula $\frac{1}{2}h(a+b)$ *for the following:*

6. $h = 7$, $a = 9$, $b = 4$.
7. $h = 9\frac{1}{2}$, $a = 12$, $b = 7\frac{1}{2}$.
8. $h = 12\frac{3}{4}$, $a = 16\frac{1}{2}$, $b = 9$.
9. $h = a = 7\frac{1}{2}$, $b = 5$.
10. $h = b = 9\frac{1}{4}$, $a = 12$.
11. $h = a = b = 12\frac{1}{2}$.

Draw the figure in Ex. 11. Find the area by a different method from that in which the formula for a trapezoid is used.

Perform the following operations:

12. $6^2 + 3 \times 8$.
13. $8^2 - 2 \times 3$.
14. $9 \div 3 + 2$.
15. $16 \div 2 - (5 - 1)$.

Evaluate the formula $3a^2 + 7(a+b)$ *for the following:*

16. $a = 5$, $b = 3$.
17. $a = 6$, $b = 2$.
18. $a = b = 4$.
19. $a = 8$, $b = 3\frac{1}{2}$.

Circle. It was shown in Book I that we can find the circumference and the area of a circle as follows:

The circumference of a circle is π (pī) times the diameter, where π is approximately $\frac{22}{7}$, or 3.1416.

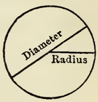
The area of a circle is π times the square of the radius.

That is, $$c = \pi d,$$
and $$A = \pi r^2.$$

If the diameter is 14 in., the radius is 7 in. Then

$$c = \pi d = \tfrac{22}{7} \times 14 \text{ in.} = 44 \text{ in.},$$

and $$A = \pi r^2 = \frac{22}{7} \times 7^2 \text{ sq. in.} = \frac{22}{\cancel{7}} \times \cancel{7} \times 7 \text{ sq. in.}$$
$$= 154 \text{ sq. in.}$$



Instead of writing $c = \pi d$, we may write $c = 2\pi r$, because $d = 2r$.

Exercise 11. The Circle

1. A certain wheel has a diameter 35 in. What is the circumference?

2. A circular lamp shade is to have a diameter of 21 in. How many inches of fringe must be purchased to go round the shade, allowing 1 in. for waste?

3. There is a circular flower bed in a park, the diameter being 14 ft. How many feet in the circumference?

4. Find the area of the flower bed in Ex. 3.

5. An iron rod has a diameter of $3\frac{1}{2}$ in. Find the area of a cross section.

This is often necessary, as in finding the strength of iron rods.

6. A tinsmith cuts 100 circular pieces of tin for the bottoms of cups, using a radius of 2 in. How many square inches of tin does he put into these circular pieces?

Cylinder. It was shown in Book I that we can find the curved surface and the volume of a cylinder as follows:

The area of the curved surface of a cylinder is the product of the height and π times the diameter.

For π times the diameter is the circumference; and if we think of the curved surface as being unrolled, we see that the circumference times the height is the curved surface of the cylinder.

The volume of a cylinder is the product of the height and the area of the base.

Expressed algebraically, these statements become

$$S = \pi d h,$$

$$V = bh.$$

The base being πr^2 we may write the second formula $V = \pi r^2 h$.

Exercise 12. The Cylinder

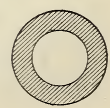
1. A cylindric water tank is 20 ft. in diameter and 10 ft. high. Find the area of the curved surface.

2. Find the capacity of the tank in Ex. 1.

3. A piece of water pipe is 10 ft. long and the internal diameter is 4 in. Find the number of cubic inches of water necessary to fill the pipe.

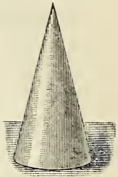
4. A cylindric iron pillar supporting a ceiling is 10 ft. high and has a diameter of 5 in. If the pillar is solid and weighs 441 lb. per cubic foot, what is the total weight?

5. A hollow cylindric iron pillar has a cross section as here shown. The external diameter is 6 in., the internal diameter 4 in., and the length 12 ft. If this grade of iron weighs 441 lb. per cubic foot, what is the total weight of the pillar?



Other Useful Formulas. It is shown in geometry that the following statements, given in Book I, are true:

The volume of a cone is one third the product of the height and the area of the base.



Expressed algebraically, this becomes

$$V = \frac{1}{3} bh.$$

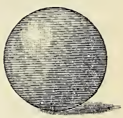
Since $b = \pi r^2$, we have

$$V = \frac{1}{3} \pi r^2 h.$$

For example, if $r = 3$ in. and $h = 7$ in. we have

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{\cancel{3}} \times \frac{22}{\cancel{7}} \times \cancel{3} \times 3 \times \cancel{7} \text{ cu. in.} = 66 \text{ cu. in.}$$

The volume of a sphere is four thirds the product of π and the third power of the radius.



That is, $V = \frac{4}{3}\pi r^3$,

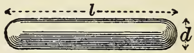
where r^3 (read "r cube") means rrr.

For example, if the radius is $3\frac{1}{2}$ in. we have

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times \frac{7}{2} \text{ cu. in.} = 179\frac{2}{3} \text{ cu. in.}$$

Exercise 13. Cone and Sphere

1. A piece of iron l inches long and d inches in diameter has a circular cross section and hemispheric ends. State a formula for the volume.



Notice that the length of the cylindric part is $l - d$.

2. Find the volume of a cone whose base has an area of $7\frac{1}{2}$ sq. in. and whose height is $9\frac{1}{2}$ in.

3. Find the volume of a cone whose base has a radius of 6 in. and whose height is 14 in.

Formulas used in Shops. If you read books on mechanics, or articles in encyclopedias about machinery, or magazines that relate to shop work of any kind, you will often find formulas used for stating rules in a brief manner.

We shall now give a few formulas used in shops, simply for the purpose of working with algebraic expressions which we may meet in practical life. Most of these formulas will be given without reference to their meaning.

Exercise 14. Formulas used in Shops

1. In the formula $W/W' = L/L'$ find the value of W/W' when $L = 6$ and $L' = 8$.

2. In the formula of Ex. 1 find the value of W when $L = 6$, $L' = 8$, and $W' = 12$.

If $\frac{1}{12}$ of W is $\frac{6}{8}$, or $\frac{3}{4}$, how much is W?

3. In the formula $M = \frac{2}{3}\pi rk$ find the value of M when $r = 9$ and $k = 14$.

4. In the formula $A = \frac{1}{4}cd$ find the value of A when $c = \frac{44}{7}$ and $d = 2$.

5. In the formula $A = \frac{1}{3}\pi kd$ find the value of A when $k = 21$ and $d = 7$.

6. In the formula $V = \frac{1}{12}\pi d^2 h$ find the value of V when $d = 7$ and $h = 26.4$.

7. In the formula $s = 16\,t^2$ find the value of s when $t = 2.4$.

8. The number of pounds which can be supported at the middle of a cylindric piece of cast-iron shafting l feet long and d inches in diameter is $500\,d^3/l$. Find the weight which can be thus supported by a piece of shafting 12 ft. long and 3 in. in diameter.

Formulas used in the Home. We can conveniently use formulas in measurements about the home. For example, the formulas which relate to areas can be used in problems about the areas of floors and walls, and those which relate to volumes can be used in problems about the capacity of various kinds of tanks, bins, and jars. The formula is easier to remember than the rule and would be used more often if more persons knew its advantages.

Exercise 15. Formulas used in the Home

1. The formula for the number of gallons in a cylindric jar of diameter d inches and height h inches is $\frac{1}{924}\pi d^2 h$. Find the number of gallons in a jar whose height is 22 in. and diameter 14 in.

2. The formula for the number of tons of hard coal in a rectangular bin l feet long and w feet wide, filled to a depth of d feet, is $\frac{1}{35}lwd$. Write this as a rule and find the number of tons of hard coal in a rectangular bin which is 10 ft. long, 6 ft. wide, and filled to a depth of 5 ft.

3. The formula for the number of gallons of water needed to fill a rectangular tank l feet long, w feet wide, and d feet deep is $\frac{15}{2}lwd$. Write this as a rule and find the number of gallons needed to fill a tank 8 ft. long, 4 ft. wide, and 3 ft. deep.

4. If a recipe calls for t tablespoons of sugar for n persons, the number of tablespoons for p persons is pt/n. What does this equal when $t = 8$, $n = 5$, and $p = 4$?

5. A woman is making a dozen circular doilies, each having a diameter of 8 in. and each having lace sewed around the edge. How many yards of lace will be needed, allowing 6 in. for joining and fulling in on each?

Formulas used in Business. Formulas are extensively used in business. While this is particularly true in insurance, investments, and construction work on buildings, it is also possible to use formulas in more common matters, as we shall see in the following problems.

Exercise 16. Formulas used in Business

1. The formula for i, the interest on p dollars for t years, the rate of interest per annum being r, is $i = prt$. Find the value of i when $p = 2000$, $t = 3$, and $r = 6\%$.

The student should be asked to explain exactly what the formula signifies and what the result means.

2. If n objects cost d dollars, what is the average price? Write this as a formula, using p for the average price.

3. In Ex. 2 find the value of p when $n = 24$ and $d = 36$.

4. In order to save d dollars in t years, how much must a man save on an average per year? per month?

5. How many packages of sugar, each package containing p pounds, can be filled from a barrel which contains b pounds of sugar? What is the result when $p = 2\frac{1}{2}$ and $b = 300$? when $p = 5$ and $b = 310$?

6. The selling price s of an article is found from the formula $s = c + p + o$, where c represents the cost, p the profit, and o the overhead charges which ought to be charged against this article. Find the value of s when $c = 38¢$, $p = 12\frac{1}{2}¢$, and $o = 2\frac{1}{2}¢$.

7. In large business concerns r, the rate of profit on sales, is found by the formula $r = p/(c + p + o)$, where the letters have the meaning given in Ex. 6. Find the value of r when $p = 25¢$, $c = 45¢$, and $o = 5¢$.

Exercise 17. Review

1. Write a rule for finding the area of each face of a cube. Express this rule as a formula, using A for the area of a face and e for the edge of the cube.

2. Find the result in Ex. 1 when $e = 17\frac{1}{2}$ in.

3. From Ex. 1 find a formula for the entire surface of a cube of edge e. What is the value when $e = 6\frac{1}{4}$ in.?

4. Write a rule for finding the number of inches in a given number of yards. Express this rule as a formula, using y for the number of yards and i for the total number of inches. Verify the result when $y = \frac{1}{2}$; when $y = 1$; when $y = 4$; when $y = 5\frac{1}{2}$.

5. If an automobile averages m miles per hour, find the distance d which it travels in t hours. Find the value of d when $m = 22\frac{1}{2}$ and $t = 3$.

6. Write a rule for finding the number of seconds in a given number of hours. Express this rule as a formula, using initial letters as usual.

7. If a man walks m miles in h hours, find the time t that it takes him to walk 1 mi. at the same rate. Find the value of t when $m = 11$ and $h = 4$.

8. State as a rule the formula found in Ex. 7.

9. If a man receives c cents per hour for regular work, and pay for time and a half when he works overtime, how much does he receive for 4 hr. overtime? How much is this when $c = 40$?

10. If a man receives wages as stated in Ex. 9, how much does he receive for r hours regular work and v hours overtime? Verify the result by letting $r = 8$, $c = 40$, and $v = 1$; by letting $r = 9$, $c = 40$, and $v = 4$.

11. Draw six right triangles, measure the sides of each, and then tabulate the results as shown in the following:

Hypotenuse of Triangle	Other Sides of Triangle		Square of Hypotenuse	Sum of Squares of Other Sides
5	3	4	25	$9 + 16 = 25$
$7\frac{1}{2}$	$4\frac{1}{2}$	6	$56\frac{1}{4}$	$20\frac{1}{4} + 36 = (\quad)$
$8\frac{1}{3}$	5	$6\frac{2}{3}$	$(\quad)$	$25 + (\quad) = (\quad)$
. . .	. . .	. . .	. . .	. . . .

From the results what relation among the three sides of a right triangle may be inferred to exist?

The student is familiar with this fact from his work in Book I.

12. If a and b are the two sides of a right triangle and h is the hypotenuse, express by means of a formula the relation found in Ex. 11.

By substituting in the formula of Ex. 12, determine which of the triangles with the following sides are right triangles:

13. 6, 8, 10.

14. $2\frac{1}{2}$, $1\frac{1}{2}$, 2.

15. $5\frac{1}{4}$, 5, $7\frac{1}{4}$.

16. 15, 8, 17.

17. 12, 13, 15.

18. 29, 20, 21.

19. By substituting any value for x in $x^2 - 1$, $2x$, and $x^2 + 1$, show that the three numbers which result are sides of a right triangle.

20. If A, B, C represent the number of degrees in the respective angles of a triangle, we know that $A + B + C = 180$; that is, the sum of the angles of any triangle is 180°. If $B = 45$ and $C = 70$, what is the value of A?

21. Find how much greater $2a + b$ is than $2a - b$ when $a = 15$ and $b = 2\frac{1}{2}$.

22. Express in words the meaning of $2(13 - 4\frac{1}{2})$.

23. Express in words the meaning of $\frac{1}{2}(a+b)$, and find the value of the expression when $a=7\frac{1}{2}$ and $b=13\frac{1}{2}$.

24. If $a=7$ and $b=5$, find the value of $(a+b)^2$ and the value of $a^2+2\,ab+b^2$. Compare these two values.

25. Compare the values of $(a+b)^2$ and $a^2+2\,ab+b^2$ when $a=3$ and $b=5$; when $a=5$ and $b=5$; when $a=9$ and $b=3$. What do you infer as to the probable equality of $(a+b)^2$ and $a^2+2\,ab+b^2$ for any other values of a and b? Try another pair of values.

26. Compare the values of $(a-b)^2$ and $a^2-2\,ab+b^2$ when $a=5$ and $b=2$; when $a=8$ and $b=1$; when $a=10$ and $b=5$. What inference do you draw? Test the accuracy of your inference for any other values of a and b, taking a greater than b.

27. Find the value of $(a+b)(a-b)$ when $a=10$ and $b=2$; when $a=8$ and $b=3$.

28. Compare the values of a^2-b^2 and $(a+b)(a-b)$ when $a=4$ and $b=1$; when $a=6$ and $b=2$; when $a=10$ and $b=4$. What inference do you draw? Test the accuracy of your inference for any other values of a and b, taking a greater than b.

29. When $a=2$ and $b=1$ compare the values of $(a+b)^3$ and $a^3+3\,a^2b+3\,ab^2+b^3$ and for any other pair of values you may care to take. What inference do you draw?

The expression $(a+b)^3$ means $(a+b)(a+b)(a+b)$.

30. When $a=3$ and $b=1$ compare the values of $(a-b)^3$ and $a^3-3\,a^2b+3\,ab^2-b^3$ and for any other pair of values you may care to take. What inference do you draw?

31. From the result of Ex. 29 find the cube of $10+2$.

32. From the result of Ex. 30 find the cube of 8 by considering it as the cube of $10-2$.

II. THE EQUATION

Need for Equations. You have seen that formulas are very useful in business life. Algebra has, however, another value connected with formulas; it shows us how we can deduce a new formula from one that we already have. To do this we need to know about *equations*.

It is better to begin studying equations by taking a few easy problems which are not connected with the formulas that we have already seen. We shall therefore begin with some easy and interesting problems which, while not useful in themselves, will show us how to solve some other problems which are of importance.

Unknown Quantity. If we see on the blackboard a statement of equality involving the sum of two numbers, like

$$2 + 7 = 9,$$

and one of the numbers is then covered by a piece of paper, thus:

$$2 + \square = 9,$$

we can easily tell what number is covered. For example, you can tell at once the numbers covered in these statements:

$$3 + \square = 12,$$

$$\square + 7 = 11.$$

The number covered is called the *unknown quantity*.

We represent the unknown quantity by a letter, usually the letter x, unless we have some special reason, such as the desire to use an initial letter, for using another. For example, you can easily determine the value that x must have in each of these statements:

$$2 + x = 5 \qquad 3 + x = 10 \qquad x + 7 = 9$$

Exercise 18. Easy Problems

All work oral

State what numbers should be written in these squares:

1. $3 + \square = 5$.
2. $5 + \square = 9$.
3. $8 - \square = 5$.
4. $9 - \square = 7$.
5. $7 + \square = 15$.
6. $\square - 6 = 10$.
7. $\square + 8 = 10$.
8. $\square - 9 = 11$.
9. $\square + 7 = 20$.
10. $\square - 5 = 25$.
11. $2 \times \square = 8$.
12. $\square \div 2 = 4$.
13. $5 \times \square = 40$.
14. $50 \div \square = 10$.
15. $\square \div 9 = 20$.

State what number x represents in each case:

16. $3x = 15$.
17. $4x = 16$.
18. $7x = 49$.
19. $\frac{1}{2}x = 1$.
20. $\frac{1}{3}x = 6$.
21. $\frac{1}{4}x = 2$.
22. $x + 3 = 5$.
23. $x - 3 = 5$.
24. $x - 9 = 1$.

25. What is the number which when multiplied by 2 is equal to 16?

26. The product of two numbers is 20 and one of the numbers is 4. What is the other number?

27. If 27 is divided by a certain number the result is 9. What is the number?

28. Half a certain number is 15. What is the number?

29. What number increased by 5 is equal to 9? In the equation $x + 5 = 9$ what is the number x?

30. What is the number which when added to 10 gives 25 as the sum? Express the question as an equation.

31. What question is naturally suggested by the equation $x + 8 = 18$? State the answer to that question.

The teacher will see that the purpose of this chapter is to lead the students to solve easy types of equations by common-sense, informal methods.

The Arte

as their workes doe extende) to diſtincte it onely into twoo partes. Whereof the firſte is, *when one nomber is equalle vnto one other.* And the ſeconde is, *when one nomber is compared as equalle vnto.2.other nombers.*

Alwaies willyng you to remēber, that you reduce your nombers, to their leaſte denominations, and ſmalleſte formes, before you procede any farther.

And again, if your *equation* be ſoche, that the greateſte denomination *Coßike*, be ioined to any parte of a compounde nomber, you ſhall tourne it ſo, that the nomber of the greateſte ſigne alone, maie ſtande as equalle to the reſte.

And this is all that neadeth to be taughte, concernyng this woorke.

Howbeit, for eaſie alteratiō of *equations*. I will propounde a fewe exāples, bicauſe the extraction of their rootes, maie the more aptly bee wroughte. And to auoide the tedious repetition of theſe woordes: is equalle to: I will ſette as I doe often in woorke vſe, a paire of paralleles, or Gemowe lines of one lengthe, thus: ======, bicauſe noe.2. thynges, can be moare equalle. And now marke theſe nombers.

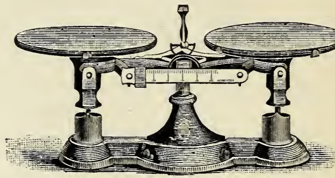
1. 14.ze.—+—.15.ϱ=====71.ϱ.
2. 20.ze.———.18.ϱ=====.102.ϱ.
3. 26.ʒ—+—10ze====9.ʒ——10ze—+—213.ϱ.
4. 19.ze—+—192.ϱ====10ʒ—+—108ϱ——19ze
5. 18.ze—+—24.ϱ.====8.ʒ.—+—2.ze.

THE FIRST PRINTED SIGN OF EQUALITY

An interesting page from one of the earliest English algebras. It shows the sign of equality as it first appeared in a printed work. The book was called *The Whetstone of Witte*, and was written by Robert Recorde, a prominent physician. It was published in London in 1557

Equation. A statement of equality between two expressions is called an *equation.*

Equations requiring Subtraction. If we place a pound weight on the right side of these scales and place 10 oz. on the left side, we see that we must add something to the left side to make the scales balance. That is,



$$10 + x = 16.$$

Here $10 + x = 16$ is an equation; $10 + x$ is called the *first member* of the equation, and 16 is called the *second member.* The two members are also called the *sides* of the equation.

We can easily see that the scales will still balance if we take 10 oz. from each side. We shall then have

$$x = 16 - 10;$$

that is, $$x = 6.$$

We have now *solved* the equation $10 + x = 16$.

How do you prove that 6 is the correct value of x?

Exercise 19. Using Subtraction

Examples 1 to 5, oral

State the value of x in each of the following equations:

1. $x + 2 = 11$.

2. $x + 7 = 12$.

3. $8 + x = 15$.

4. $9 + x = 12$.

5. $6 + x = 14$.

6. $x + 2\frac{1}{2} = 3\frac{1}{4}$.

7. $x + 3\frac{1}{4} = 5\frac{1}{8}$.

8. $2\frac{1}{4} + x = 6\frac{1}{2}$.

9. $3\frac{3}{4} + x = 5\frac{1}{2}$.

10. $2\frac{1}{2} + x = 6\frac{3}{8}$.

11. $2.3 + x = 4.2$.

12. $3.7 + x = 6.3$.

13. $x + 2.9 = 5.2$.

14. $x + 3.8 = 7.5$

15. $x + 4.6 = 8.2$.

Equations requiring Division. If a boy earns 20¢ to-day, which is twice as much as he earned yesterday, how much did he earn yesterday? How do you find the result?

If $2x = 20$, what is the value of x? How do you find it?

If $4x = 24$ and we divide one member of the equation by 4, by what must we divide the other member in order to continue to have an equation? What is the value of x? How do you prove that this is the correct value?

It is evident that no algebraic division is needed for these cases.

Exercise 20. Using Division

Examples 1 to 7, oral

1. Twice the number of which I am thinking is 16. What is the number? How do you find it?

2. If 3 times a certain number is 21, what is the number? State the equation. How do you solve it?

3. State a question suggested by the equation $7x = 21$ and state the answer to that question.

Solve each of the following equations:

4. $2x = 20$.	8. $13x = 169$.	12. $2\frac{1}{2}x = 7\frac{1}{2}$.
5. $3x = 33$.	9. $15x = 315$.	13. $3\frac{1}{3}x = 10$.
6. $4x = 28$.	10. $17x = 357$.	14. $4\frac{3}{4}x = 14\frac{1}{4}$.
7. $5x = 50$.	11. $19x = 399$.	15. $6\frac{2}{3}x = 40$.

Make a problem about each of the following equations and then solve that problem:

16. $7x = 63$.	18. $27x = 81$.	20. $2.7x = 10.8$.
17. $9x = 63$.	19. $35x = 245$.	21. $2.7x = 1.08$.

The simplest types of equations are those requiring either subtraction or division, and these are given on page 30 and on this page.

Equations requiring Addition. If you spend 10¢ at a store and have 15¢ left, how much had you at first?

If $x - 10 = 15$, what is the value of x?

In the equation $x - 10 = 15$, if you add 10 to the second member, what must you also add to the first member in order to continue to have an equation?

What is the value of $x - 10 + 10$?

What is the value of $15 + 10$?

We see that we solve the equation in this way:

Given $$x - 10 = 15,$$

we add 10 to each member and then have

$$x - 10 + 10 = 15 + 10,$$

or $$x = 25,$$

because $x - 10 + 10$ is the same as x, and $15 + 10 = 25$.

Exercise 21. Using Addition

Examples 1 to 6, oral

1. If a boy has 16 stamps left after using 5, how many had he at first? State the equation. How do you solve it?

2. I am thinking of a number which when decreased by 7 becomes 5. What is the number? State the equation. How do you solve the equation?

Solve each of the following equations:

3. $x - 2 = 7$.	**7.** $x - 27 = 39$.	**11.** $x - 2\frac{1}{2} = 3\frac{3}{4}$.
4. $x - 6 = 6$.	**8.** $x - 49 = 56$.	**12.** $x - 3\frac{1}{4} = 4\frac{3}{8}$.
5. $x - 7 = 12$.	**9.** $x - 139 = 47$.	**13.** $x - 5\frac{7}{8} = 2\frac{3}{4}$.
6. $x - 9 = 21$.	**10.** $x - 276 = 398$.	**14.** $x - 7\frac{3}{8} = 3\frac{7}{8}$.

The next equations in point of simplicity, after those treated on pages 30 and 31, are those involving addition and multiplication.

Equations requiring Multiplication. If half of a class goes to the blackboard and there are 10 who go to the blackboard, how many students are there in the class?

If $\frac{1}{2}x = 10$, what is the value of x? How do you find it?

If you multiply the first member of the equation

$$\frac{1}{2}x = 10$$

by 2, what must you do to the second member in order to continue to have an equation?

We solve the equation $\frac{1}{5}x = 7$ as shown below:

Given $\frac{1}{5}x = 7,$

we multiply each member by 5 and then have

$$x = 35.$$

How do you prove that 35 is the correct value?

Exercise 22. Using Multiplication

Examples 1 to 7, oral

1. If someone is thinking of a number the fourth part of which is 6, what is the number? State the equation.

2. State a problem suggested by the equation $\frac{1}{3}x = 9$ and solve that equation.

Solve each of the following equations:

3. $\frac{1}{2}x = 9.$

4. $\frac{1}{4}x = 9.$

5. $\frac{1}{4}x = 7.$

6. $\frac{1}{5}x = 11.$

7. $\frac{x}{7} = 10.$

8. $\frac{1}{8}x = 16.$

9. $\frac{1}{8}x = 1.6.$

10. $\frac{1}{8}x = 1\frac{4}{5}.$

11. $\frac{1}{5}x = 2\frac{1}{2}.$

12. $\frac{x}{8} = 2\frac{1}{2}.$

13. $\frac{1}{6}x = 3\frac{1}{3}.$

14. $\frac{1}{10}x = 0.9.$

15. $\frac{1}{12}x = 2.4.$

16. $\frac{1}{16}x = 2\frac{1}{2}.$

17. $\frac{x}{24} = 2\frac{1}{2}.$

Solution of Equations. We are now ready to solve equations the solutions of which require some combination of the different methods already studied.

1. Solve the equation $\frac{3}{4}x = 7$.

If this were $2x = 7$, we know that we should simply divide each member by 2. We proceed in a similar way with the given equation and divide each member by $\frac{3}{4}$. We then have

$$\begin{aligned} x &= 7 \div \tfrac{3}{4} \\ &= 7 \times \tfrac{4}{3} = \tfrac{28}{3} = 9\tfrac{1}{3}. \end{aligned}$$

2. Solve the equation $\dfrac{2\frac{1}{2}x}{3} = 8$.

Multiplying each of these equal expressions by 3, we have

$$2\tfrac{1}{2}x = 24,$$

or

$$\tfrac{5}{2}x = 24.$$

Dividing each of these equal expressions by $\frac{5}{2}$, we have

$$\begin{aligned} x &= 24 \div \tfrac{5}{2} \\ &= 24 \times \tfrac{2}{5} = \tfrac{48}{5} = 9\tfrac{3}{5}. \end{aligned}$$

3. Solve the equation $\dfrac{7.2}{x} = 8.3$.

Multiplying each of these equal expressions by x, we have

$$7.2 = 8.3\,x.$$

Dividing each of these equal expressions by 8.3 and noticing that the equation $\dfrac{7.2}{8.3} = x$ may be written $x = \dfrac{7.2}{8.3}$, we have

$$x = \frac{7.2}{8.3} = \frac{72}{83} = 0.867\ldots,$$

where the dots after 0.867 mean the division can be carried farther.

In a case like this it is customary to leave the result in the form of a common fraction if the denominator is 2, 3, 4, 5, 8, 12, 16, 32, or 64, because such common fractions, and also $\frac{1}{6}$, $\frac{1}{9}$, and $\frac{1}{10}$, are often used in business. In other cases, as in this one, it is best to reduce the result to a decimal, carrying it to three places.

Exercise 23. Solution of Equations

Examples 1 to 7, oral

1. If five times a certain number is 45, what is the number? State the equation. How do you solve it?

2. If the quotient found by dividing 35 by a certain number is 5, what is the number? State the equation. How do you solve it?

Solve the following equations:

3. $5x = 5$.	**8.** $\frac{2}{3}x = 1\frac{1}{3}$.	**13.** $7x = \frac{2}{3}$.
4. $5x = 2\frac{1}{2}$.	**9.** $\frac{3}{4}x = 6$.	**14.** $2\frac{1}{2}x = 12\frac{1}{2}$.
5. $3x = 1$.	**10.** $\frac{5}{8}x = 1\frac{1}{4}$.	**15.** $3\frac{7}{8}x = 31$.
6. $\frac{2}{3}x = \frac{2}{3}$.	**11.** $\frac{7}{8}x = 3\frac{1}{2}$.	**16.** $7.9x = 31.6$.
7. $\frac{3x}{10} = 10$.	**12.** $\frac{12}{x} = 2\frac{2}{5}$.	**17.** $\frac{3.6}{x} = 7$.

18. If four times a certain number is divided by 7, the result is 8. What is the number?

19. If five times a certain number is divided by 2.7, the result is 3. What is the number?

20. The number of inches in the length of a certain rectangle is 9 and the number of square inches in the area of the rectangle is 36. Find the width of the rectangle.

In writing equations and expressions in algebra the labels of the numbers, like "inches" and "square inches," are usually omitted, it being assumed that the proper units are understood.

21. The area of a certain rectangle is 10.8 and the length is 9. Find the width.

22. The area of a certain triangle is 17.6 and the base is 8. Find the height.

23. Solve Ex. 22 for $A = 23$, $b = 5\frac{3}{4}$.

Harder Equations. The following should be studied:

1. Solve the equation $2x - 3\frac{1}{2} = 9\frac{3}{4}$.

Adding $3\frac{1}{2}$ to each of these equal expressions, so as to avoid $3\frac{1}{2}$ in the first member, we have

$$2x - 3\tfrac{1}{2} + 3\tfrac{1}{2} = 9\tfrac{3}{4} + 3\tfrac{1}{2},$$

or $$2x = 13\tfrac{1}{4}.$$

Dividing by 2, $$x = 6\tfrac{5}{8}.$$

2. Solve the equation $7.8 - 4x = 5.2$.

Adding $4x$ to each of these equal expressions, we have

$$7.8 = 5.2 + 4x.$$

Subtracting 5.2, $$2.6 = 4x,$$

or $$4x = 2.6.$$

Dividing each of these equal expressions by 4, we have

$$x = 0.65.$$

Exercise 24. Solution of Equations

1. If 6 is added to five times a certain number, the result is 71. What is the number?

2. If 7 is subtracted from three times a certain number, the result is 38. What is the number?

Solve the following equations:

3. $3x + 8 = 53$.

4. $4x - 7 = 61$.

5. $9 + 7x = 44$.

6. $\frac{1}{2}x + 6 = 18$.

7. $\frac{2}{3}x - 10 = 0$.

8. $\frac{5}{8}x + 6 = 21$.

9. $3.42x - 0.84 = 6$.

10. $5.78x + 1.1 = 30$.

11. $8.26x - 8.06 = 0.2$.

12. $36.8 - 4.2x = 28$.

13. $\frac{2}{3}x - 19.08 = 0.92$.

14. $3.25x + 7.3 = 33.3$.

Equations and Checks. The following should be studied:

1. Solve the equation $2(4x + 9) = 42$.

Dividing each member of the equation by 2, we have

$$4x + 9 = 21.$$

Subtracting 9 from each member, we have

$$4x = 21 - 9 = 12.$$

Dividing each member by 4, we have

$$x = 3.$$

We prove that this is the correct solution by putting 3 in place of x in the original equation, thus:

$$2(4 \times 3 + 9) = 2(12 + 9) = 2 \times 21 = 42,$$

so we see that the two members of the equation are equal.

2. Solve the equation $\dfrac{3x - 2}{6\frac{1}{2}} = 2$.

Multiplying each member by $6\frac{1}{2}$, we have

$$3x - 2 = 13.$$

Adding 2 to each member, we have

$$3x = 15.$$

Dividing each member by 3, we have

$$x = 5.$$

When we find whether our work is correct we are said to *check* it. As we have already seen, we check the value of the unknown quantity by putting that value in place of the letter in the equation. If the two members of the equation are then equal, we know that the result is correct.

For example, in the preceding equation we put 5 in place of x, and we have

$$\frac{3 \times 5 - 2}{6\frac{1}{2}} = \frac{15 - 2}{6\frac{1}{2}} = \frac{13}{6\frac{1}{2}} = 2,$$

so that we have checked the result.

Exercise 25. Solution of Equations

Examples 1 to 7, oral

Solve the following equations:

1. $2(x-1)=4$.
2. $3(x+1)=9$.
3. $4(x-2)=16$.
4. $5(x+2)=25$.
5. $5(x-7)=35$.
6. $6(x+7)=48$.
7. $7(3+x)=42$.
8. $\dfrac{2x+1}{3}=5$.
9. $5(2x+3)=55$.
10. $7(3x-1)=56$.
11. $9(5x+7)=108$.
12. $2\frac{1}{2}(3x-4)=50$.
13. $\frac{5}{8}(12x+7)=19\frac{3}{8}$.
14. $\frac{3}{8}(16x-5)=1\frac{1}{8}$.
15. $3\frac{1}{2}(5x+12)=217$.
16. $\dfrac{2x+5}{7}=21$.

17. If 3 is added to seven times a certain number and the sum divided by 9, the quotient is 5. Find the number.

What does $\dfrac{7x+3}{9}$ equal? How do you solve the equation?

18. If $\frac{2}{3}x$ is increased by 9 and the result divided by $8\frac{1}{2}$, the quotient is 2. What is the value of x?

Solve the following equations and check the results:

19. $\dfrac{5x-7}{2}=14$.
20. $\dfrac{8-5x}{1\frac{1}{2}}=2$.
21. $\dfrac{12.1x-3}{10.6}=2$.
22. $\dfrac{8x+3}{3}=23$.

23. A person asks you to think of a whole number larger than 3; to multiply it by 7; to take away 4; to divide the result by 12; and then to tell him the quotient. If you say the quotient is 2, he can tell you that the number you thought of was 4. How does he know it?

Formulas found from Other Formulas. The purpose of the work given on pages 27–38 is chiefly to show another important use of algebra. You probably found the work interesting in itself, because it brought accurate results and because it showed you how to solve many little puzzles with numbers. The real object, however, is to prepare you to find formulas for yourselves when certain other formulas are given. We shall now consider a few simple cases.

1. If you know that the area of a rectangular field is 1 acre, and have measured the width of the field, how can you find the length without measuring?

Since we know, from page 10, that

$$A = lw,$$

and know the width, we may divide each member by w and have

$$\frac{A}{w} = l.$$

That is, we have a formula for l in terms of A and w.

Thus, if $A = 1$ acre $= 160$ sq. rd. and if $w = 10$ rd., we have

$$l = \frac{A}{w} = \frac{160}{10}\text{ rd.} = 16\text{ rd.}$$

2. From the formula for the area of a triangle, $A = \frac{1}{2}bh$, find the value of h.

Multiplying each member of the equation by 2, we have

$$2A = bh.$$

Dividing each member by b, we have

$$\frac{2A}{b} = h.$$

That is, we have a formula for the height of a triangle in terms of the area and the base.

Thus, if $A = 175$ and $b = 17$, then

$$h = \frac{2A}{b} = \frac{2 \times 175}{17} = 2 \times 10\tfrac{5}{17} = 20\tfrac{10}{17} = 20.588.$$

Exercise 26. Formulas

1. Given the formula $V = lwh$, found on page 12, find a formula for l in terms of V, w, and h.

2. Given the formula $B = \frac{1}{12} lwt$, found on page 12, find a formula for l in terms of B, w, and t. Apply this formula to finding l when $B = 24$, $w = 8$, and $t = 2$.

3. Given the formula $A = \frac{1}{2} h (a + b)$, found on page 16, find a formula for a in terms of A, h, and b.

If you first multiply each member of the formula equation by 2, what is the resulting equation? If you then divide each member by h, what is the resulting equation? What must now be done to find a formula for a? What is the result?

4. In Ex. 3 find a formula for b in terms of A, h, and a.

5. From the results of Exs. 3 and 4 write two rules relating to the trapezoid.

6. Given the formula $A = \frac{1}{2} h (a + b)$, what is the value of a when $A = 34$, $h = 4$, and $b = 7$?

7. Given the formula $A = \frac{1}{2} h (a + b)$, what is the value of h when $A = 136$, $a = 10$, and $b = 7$?

8. Given the formula $c = \pi d$, found on page 18, what is the value of d in terms of c and π?

9. If $c = \pi d$, find the value of d when $c = 22$.

Remember that π always stands for a certain number which is approximately $\frac{22}{7}$, so that we use $\frac{22}{7}$ for π in all simple computations.

10. If $c = \pi d$, find the value of d when $c = 6.6$.

11. If $c = 2 \pi r$, find a formula for r.

12. Given the formula $S = \pi dh$, found on page 19, find a formula for h. If $S = 66$ and $d = 7$, find the value of h.

Teachers may omit such parts of pages 41 and 42 as are not within the range of interest of the class.

13. Write a rule which explains the meaning of the formula $S=\pi dh$, found on page 19, and a rule for finding d when S and h are given.

14. Given the formula $V=bh$, found on page 19, derive two other formulas from this formula, and then write the results in the form of rules.

15. Given the formula $V=\frac{1}{3}hb$, found on page 20, derive two other formulas from this formula, and then write the results in the form of rules.

16. Given the formula $V=\frac{1}{3}\pi r^2 h$, found on page 20, write a rule which explains the meaning of the formula.

17. In Ex. 16 derive a formula for h and then write a rule which explains the meaning of the formula.

18. In Ex. 17, if $V=51\frac{1}{3}$ and $r=7$, find the value of h.

19. Given the formula $W/W'=L/L'$, find a formula for W in terms of the other three letters.

In Exs. 19–24 the meaning of each of the various formulas is not given, but each formula is used in practical work and the student is certain to meet with cases of this kind.

20. Given the formula $M=\frac{2}{3}\pi rk$, find a formula for r.

21. Given the formula $V=\frac{1}{12}\pi d^2 h$, find a formula for h.

22. Given the formula $A=p(1+rt)$, find a formula for r and one for t.

If you first divide each member by p, what is the result? What should you do next? What next?

23. A machinist using a cutting lathe finds in a book issued by the manufacturers the formula $n=3.8\,s/d$. From this he wishes to find a formula for d. Find that formula.

24. Given the formula $\frac{W}{P}=\frac{2R}{R-r}$, compute the value of P when $R=3.75$, $r=2.4$, and $W=1250$.

25. The freezing point of water on the centigrade scale is 0° and the boiling point is 100°. On the Fahrenheit scale these points are respectively 32° and 212°. The formula $C = \frac{5}{9}(F - 32)$ shows how to express any number of degrees (F) on the Fahrenheit scale in degrees (C) on the centigrade scale. Find the value of C when $F = 70$, that is, when the temperature as recorded by our common thermometer is 70°.

26. In Ex. 25 find the value of C when $F = 212$; when $F = 100$; when $F = 32$.

Remember that the product of zero and any other number is zero.

27. From the formula of Ex. 25 find a formula for F in terms of C.

28. A workman finds in a book on shop mathematics the formula $T = D - 1.3/N$. From this he wishes to find a formula for D in terms of T and N. What is the formula?

Add $1.3/N$ to each member of the equation.

29. There is a formula $L = \frac{13}{8}(D + d) + 2C$ which is used in connection with the length of belting in a machine shop. From this find a formula for C.

30. In making gasoline engines the formula $H = D^2N/2.5$ is used. From this find a formula for N.

31. In computing power in mechanics there is used the formula $k = Wv^2/2g$. From this find a formula for W.

32. In Ex. 31 find a formula for g.

33. From the formula $r = 3 + \frac{1}{6}s$ find a formula for s.

34. Given the formulas $A = \pi r^2$ and $c = 2\pi r$, find a formula for A in terms of c.

Since $c = 2\pi r$, we have $r = \frac{c}{2\pi}$. We may therefore put $\left(\frac{c}{2\pi}\right)^2$ for r^2 in the formula $A = \pi r^2$.

Exercise 27. Miscellaneous Problems

1. The formula $S = \frac{1}{2}gt^2$ is used to find the distance through which a body will fall in a given time. In the formula S is the distance in feet, g is equal to 32.15, and t is the time in seconds. How far will a body fall in 5 sec.?

2. A rectangular excavation is to be made w feet wide and d feet deep. The earth weighs p pounds per cubic foot. How many tons of earth will be removed in l feet of length? Evaluate the result for $w = 34$, $d = 18$, $l = 600$, $p = 70$.

3. The number of pounds in the weight of a grindstone is found by multiplying the square of the diameter in inches by the thickness in inches, and this product by 0.06363. Express this as a formula.

4. Using the formula found in Ex. 3, compute the weight of a grindstone 32″ in diameter and 4.5″ thick. How much has the weight been decreased when this stone has been worn down 2.5″ in diameter?

5. The weight of 1 cu. ft. of water is 62.5 lb. Write a formula for the weight of the water that will fill a rectangular tank l feet long, w feet wide, and h feet deep.

6. A given volume of copper weighs 8.9 times as much as the same volume of water. Using the weight of water given in Ex. 5, find a formula for the weight of l feet of copper wire having a diameter of d inches.

7. Given the formula $F = Wh/L$, find a formula for W; for h; for L.

8. The formula for the radius of a circle in terms of the circumference is $r = c/2\pi$. Write this as a rule.

9. If the circumference of an iron pipe is c inches, the diameter is c/π inches, or $d = c/\pi$. Write this as a rule.

10. A high-school boy who is studying more advanced algebra knows the values of S, n, and a in the formula $S = \frac{1}{2}n(a+l)$. How can he find the value of l?

11. Letting w represent the number of weeks in a year, s the number of weeks of school, and v the number of weeks of vacation, write an equation expressing a relation of these values. Substitute the proper values for your own school and thus check your work.

Express Exs. 12–16 as equations and solve each equation:

12. A number added to 4 times itself is equal to 45.

13. A number multiplied by 18 is equal to 126.

14. A merchant marked an article d dollars and sold it at a discount of 20%. The selling price was \$8.80.

15. A number increased by 7 times itself is equal to 168.

16. The amount of p dollars drawing interest at the rate of $r\%$ per annum for t years is A. Solve for p.

17. A workman reads in a book on mechanics that the relation between the standard size of a bolt and the bolt head is determined by the equation $f = 1.5\,d + 0.125$. If he knows the value of f, how can he find the value of d?

18. If a girl knows the values of l, a, and n in the equation $l = a + (n-1)\,d$, how can she find the value of d?

19. Check the formula found as a result in Ex. 18 by substituting certain numbers for the letters.

20. Air is 0.001276 times as heavy as an equal volume of water, and 1 cu. ft. of water weighs 62.5 lb. Compute the weight of the air in your classroom. Write a formula to express the weight of the air in a room l feet long, w feet wide, and h feet high.

21. A mechanic reads that the taper of keys is computed by the formula $T = 12(a - b)/L$. From this derive an equation having only L in the first member.

22. A given volume of ice weighs 0.92 times as much as the same volume of water, and 1 cu. ft. of water weighs 62.5 lb. Find a formula for giving the weight of a piece of ice l feet long, w feet wide, and t inches thick.

23. Some engineers state that the cross-section area of a cold-air box of a furnace should be equal to the combined area of all the registers. In a house there are to be 14 registers, each $10'' \times 14''$. What should be the cross-section area of the cold-air box?

24. Express a formula that might be used for computations such as that in Ex. 23.

25. Some lumbermen use this rule for finding in pounds the weight of rough timber. Multiply the length in feet by the breadth in feet by the thickness in inches, and multiply this product by the proper one of the following factors; for oak, 4.04; for elm, 3.05; for white pine, 2.97; for yellow pine, 3.44. Write four formulas which shall express the weight of a stick of timber of each of these four kinds of wood.

26. Using the formulas just obtained, find the weight of a stick of timber of each of the kinds of wood given in Ex. 25, the stick being $2' \times 18' \times 4.5''$.

27. Given the formula $W/P = 2R(R - r)$, compute the value of P when $R = 3.25$, $r = 2.3$, and $W = 1200$.

Teachers are advised to select from pages 43–45 only such problems as are easily understood by the students. Although the problems are not difficult, they lose much of their value and interest if their significance is not apparent. The problems that are not required may be considered as optional.

Exercise 28. Review

Examples 1 to 8, oral

Solve each of the following equations:

1. $7x + 4 = 25$.
2. $3x - 3 = 42$.
3. $\frac{1}{2}x + 4 = 7$.
4. $\frac{2}{3}x = 12$.
5. $3n + 11 = 12$.
6. $5n - 9 = 23$.
7. $2(3x + 1) = 20$.
8. $3x + 5x + 2x - 3 = 17$.

9. Given the formula $LF/W = h$, find a formula for W in terms of the other three letters.

10. If 323 is divided by a certain number, the result is 19. What is the number?

11. Given the formula $F = WP/2\pi r$, find a formula for P in terms of the other letters.

12. If to 4 times a certain number we add 13, the sum is 61. What is the number?

13. If 14 is subtracted from 6 times a certain number, the result is 122. What is the number?

14. Given that $4.75x + 3.25 = 55.5$, find the value of x.

15. Given that $\frac{3}{7}x - 4.5 = 28.5$, find the value of x.

16. Given the formula $T = D - 1.3/N$, find a formula for N in terms of the other letters.

17. Given the formula $V = \frac{1}{3}\pi r^2 h$, find a formula for r^2 in terms of the other letters.

18. A pump delivers g gallons of water per stroke and is set for s strokes per minute. Find a formula from which the number of gallons delivered per hour may be computed. Find a formula from which the number of barrels of water delivered per hour may be computed, given that 1 bbl. = 31.5 gal.

III. THE GRAPH

Value of the Graph. At the present time in statistics and in many kinds of business we see numbers represented by pictures or diagrams of some kind. The general name given to a picture which represents numbers, relative sizes, or other mathematical facts is *graph.*

The practical value of the graph may be easily seen in this illustration. Five high schools are playing a series of games of basket ball with the following results up to date :

School	Won	Lost
A	9	1
B	7	3
C	6	4
D	3	7
E	0	10

While it is evident which team has the best record, the standing of the teams is more clearly seen by the aid of the graph. Here each lightly shaded square represents a game won and each black square represents a game lost.

Bar Pictogram. Because we represent the numbers by bars, this kind of graph is often called a *bar pictogram.* It is one of the most common graphs and is frequently seen in newspapers and magazines.

Squared Paper. Graphs are conveniently represented on paper ruled in squares. Such paper is called *squared paper.*

Squared paper is also called *coordinate paper.* Students should be supplied with such paper. It can be purchased in sheets of various sizes at any stationer's. It is sometimes ruled in tenths of an inch, sometimes in eighths of an inch, and sometimes in millimeters.

Exercise 29. Bar Pictograms

By means of bar pictograms represent the following:

1. In a certain year the four teams standing highest in baseball in one of the leagues had the following records: A, won 86, lost 66; B, won 87, lost 67; C, won 86, lost 67; D, won 81, lost 72.

When the numbers are as large as this it is convenient to take each square as representing 10 games. We shall then have, for team A, 8.6 squares for games won, and we can estimate the 0.6 with the eye closely enough in this case for all practical purposes.

2. In a certain year the four teams standing highest in one of the baseball leagues had the following records: W, won 86, lost 50; X, won 85, lost 53; Y, won 72, lost 67; Z, won 69, lost 69.

Members of the class who are interested in the records of any series of games should be asked to prepare bar pictograms to show the relative standings of the teams.

3. The total ordinary receipts of the United States government in tens of millions of dollars, for ten consecutive years, expressed to the nearest $10,000,000, were 59, 66, 60, 60, 68, 70, 69, 72, 73, 70.

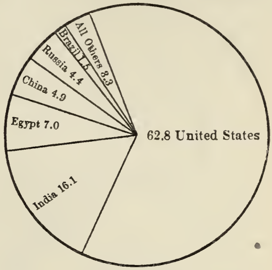
Represent the years by the figures 1, 2, 3, and so on, to 10.

4. The number of millions of long tons of cane sugar produced in the world during ten consecutive years was 4.6, 6.7, 7.3, 6.9, 7.6, 8.3, 8.4, 9.1, 9.2, 9.9.

5. The expense of our post-office system for ten consecutive years, in hundreds of millions of dollars, was as follows: 1.8, 1.9, 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.8, 3.

6. The expense of our rural-delivery service for ten consecutive years, in millions of dollars, was as follows: 25, 27, 34, 36, 37, 37, 42, 46, 47, 50.

Circular Pictogram. In a recent year the United States produced 62.8% of the cotton used for making cloth; India produced 16.1%; and other countries produced various amounts. These statistics may be represented by a circular graph as here shown.



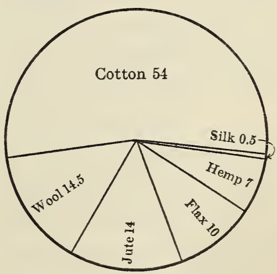
Such a graph is called a *circular pictogram.*

The circular pictogram is not so easily made as the bar pictogram, because it is more difficult to measure or estimate the arcs or angles than it is to measure or estimate the lengths of the bars in a bar pictogram. It is more valuable, however, when per cents are used.

If the students have protractors the teacher may require that the angles be measured; otherwise the students may estimate the angles.

Exercise 30. Circular Pictograms

1. This pictogram shows in per cents the relative amounts of materials used in the world in a recent year in the making of articles of clothing. Write a statement giving the facts as shown in the pictogram.



2. In a certain school 10% of the pupils are in the first grade, 16% in the second, 17% in the third, 15% in the fourth, 14% in the fifth, 11% in the sixth, 10% in the seventh, and 7% in the eighth. Represent this by a circular pictogram.

3. Large corporations usually have to compute cost to a small fraction of a cent. The pictogram given below shows the statement of a certain street railway company as to where each nickel of income went during a recent year.

Write a statement of about half a page telling what the pictogram shows as to how the income was spent.

4. A man invests 33% of his money in bonds, 12% in stocks, 30% in real estate, and the balance in his business. Represent all this by a circular pictogram.

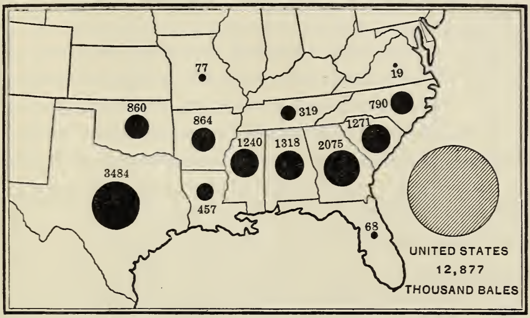
5. Of the total value of the exports in a recent year from the United States to Cuba, Porto Rico, Hawaii, and the Philippines, 50% went to Cuba, 20% to Porto Rico, 13% to Hawaii, and 17% to the Philippines. Represent all this by a circular pictogram.

6. A manufacturing concern devotes 60% of its gross income to wages, 12% to office expenses, 8% to advertising, 11% to selling expenses, 5% to dividends, 2% to interest, and the balance to incidental expenses. Represent all this by a circular pictogram.

The students may be asked to ascertain the enrollment in the different classes and represent the per cents by a circular pictogram.

Cartogram. A convenient aid to the eye is a map on which the statistics are represented graphically. Such a map is called a *cartogram.*

The following cartogram shows the average annual production of cotton in thousands of bales over a recent period of 10 yr. It shows by means of the black circles



the relative production of cotton in the cotton-producing states compared with the total production of the United States, which is indicated by the shaded circle at the right.

Exercise 31. Cartograms

1. Draw a rough map of the city or village in which you live. Shade differently the commercial section, the manufacturing section, and the residential section.

2. Draw a map as in Ex. 1 and shade differently the most densely populated section, the section of medium density, and the section which is thinly populated.

3. Draw a map of the following states and represent by circles the relative populations in millions: Ohio, 5; Indiana, 3; Illinois, 6; Michigan, 3; Wisconsin, 2.5.

Exercise 32. Review

1. The following table indicates the approximate population of several states according to the decennial census. Represent this by a bar pictogram for each state, each square representing a population of 1,000,000.

States	1870	1880	1890	1900	1910
Connecticut	537,000	623,000	746,000	908,000	1,115,000
Georgia	1,184,000	1,542,000	1,837,000	2,216,000	2,609,000
Illinois	2,540,000	3,078,000	3,826,000	4,822,000	5,639,000
Indiana	1,681,000	1,978,000	2,192,000	2,516,000	2,701,000
Massachusetts	1,457,000	1,783,000	2,239,000	2,805,000	3,366,000
Virginia	1,225,000	1,513,000	1,656,000	1,854,000	2,062,000

2. The population of the United States by the decennial census is given approximately in millions in the following table. Represent this graphically by bar pictograms, each square representing a population of 10,000,000.

Year	1800	1810	1820	1830	1840	1850	1860	1870	1880	1890	1900	1910
Millions	5	7	10	13	17	23	31	39	50	63	76	92

3. The following table shows some of the record-breaking runs of steamships between New York and Queenstown. Represent this graphically.

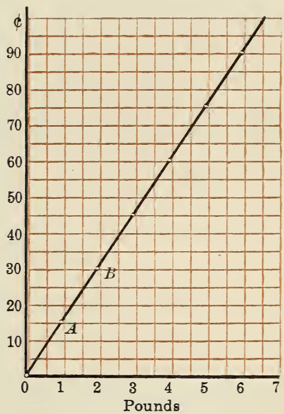
Date	1856	1876	1887	1908	1910
Time	9 da. 2 hr.	7 da. 12 hr.	6 da. 5 hr.	4 da. 15 hr.	4 da. 11 hr.

4. Write a statement showing a relation between the two maps in Exs. 1 and 2, page 51, as that the manufacturing plants cause the greatest density of population, that is, the greatest number of inhabitants per square mile.

Functional Relations. We now come to a more important kind of graph, that in which one number depends upon another. For example, the number of dollars or cents which we pay for some flour depends on the number of barrels, bags, or pounds that we buy.

We say that the cost depends on the amount purchased or that it is a *function* of the amount. This term need not be used at this time, but the idea is very important. A large part of mathematics is a study of functions, or of the functional relation of quantities. For example, the sum of 2 and some other number depends upon what that number is; the product of 3 and some other number is a function of that other number; the area of a square is a function of the side; and so on, through the whole domain of mathematics.

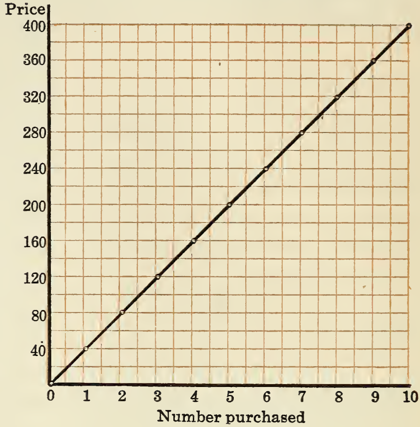
For example, this figure is a *price graph.* It tells at a glance how much any number of pounds of anything costs at 15¢ a pound. The figures at the left represent the cost, and those below represent the number of pounds. Since 1 lb. costs 15¢, we make a dot or a small circle at the letter A, above the figure 1 and on the line of 15¢. Since 2 lb. cost 30¢, we mark the point above 2 and on the line of 30¢. We might continue this for other points, but it is not necessary. Drawing a line through A and B, we have a price graph for 15¢ a pound.



We have here used two spaces for 1 lb. and one cost space for 5¢, so as to make the figure easily seen.

The student should study this graph sufficiently to see clearly how it works, finding the cost of 4 lb., 5 lb., $5\frac{1}{2}$ lb., and so on, and seeing how graphs for other prices are made.

Diagonal Graph. We can, if we wish, use a diagonal of a square as a price graph for any prices whatever. For example, in this graph we have taken the price of one unit (pound, yard, piece, or whatever it may be) as 40 cents (dollars, dimes, or whatever price is charged for the one unit). By following up the line of $7\frac{1}{2}$ (halfway from 7 to 8) we see that it cuts the graph opposite 300 (halfway between 280 and 320), and hence $7\frac{1}{2}$ will cost \$3.



But if we wish the cost at 50¢ each, all we need to do is to erase the prices at the left, writing 50, 100, 150, 200, and so on. We shall then have a price graph at 50¢ each.

Exercise 33. Price Graphs

1. Draw a price graph of cloth at 30¢ a yard. From it find the cost of $7\frac{1}{2}$ yd.

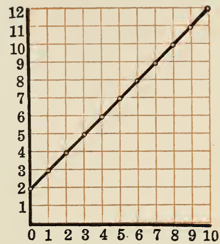
2. Draw a price graph of carpet at \$1.25 a yard. From it find the cost of 16 yd.

3. Draw a price graph of lamps at 75¢ apiece. From it find the cost of 9 lamps.

4. Draw a price graph of balls of cord at 25¢ apiece. From it find the number that can be bought for \$2.75.

Graph of a Table. We see that price graphs, like those shown on pages 53 and 54, are really graphs of multiplication tables, such as 3×15, 5×40, and so on. In the same way we may have graphs of various other kinds of tables.

For example, this is a graph of the addition table of 2's. The numbers added to 2 are shown on the lower horizontal line, and the sums are shown on the vertical line. We see that the vertical line drawn from 2 cuts the graph in the horizontal line drawn from 4; that is, the graph shows that $2 + 2 = 4$.



Such simple graphs are introduced only because they lead easily to the important kinds of graphs studied on the following pages.

Exercise 34. Graphs of Tables

1. Draw a graph of the addition table of 3's.

2. Draw a graph of the addition table of 7's, and from it find the value of $8 + 7$.

3. Draw a graph of the multiplication table of 3's.

4. Draw a graph of the multiplication table of 4's and from it find the value of 8×4.

5. From the graph of the addition table of 2's at the top of the page find the value of $8 - 2$; of $9 - 2$; of $12 - 2$.

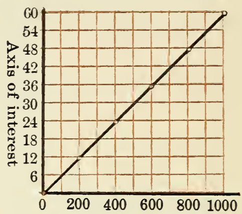
To find the value of $8 - 2$ begin at 8 on the vertical line, pass to the right until the graph is reached and note that this point is above the 6 in the horizontal row of numbers. Hence $8 - 2 = 6$.

6. From the graph of the multiplication table in Ex. 4 find the value of $36 \div 4$.

Proceed in a manner similar to that followed in Ex. 5.

Axes. In the work on pages 53–55 we have used two principal lines to represent numbers. It is convenient to have special names for these lines, and so we call them *axes*. To distinguish between the axes we call one axis the *horizontal axis* or *x-axis* and the other the *vertical axis* or *y-axis*. It is often convenient to give special names to the axes, such as the *axis of weights* and the *axis of prices*, as on page 53; the *axis of numbers* and the *axis of prices*, as on page 54; the *axis of addends* and the *axis of sums*, as on page 55, and so on, the names depending upon what the numbers represent.

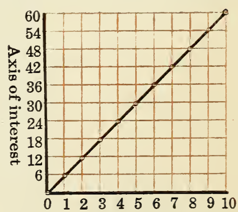
Interest Graph. If we wish to find the interest on any sum for 1 yr. at 6%, we see that $i = 6\%$ of p. The graph will therefore be merely that of the multiplication table of 6%, as here shown. We see at a glance that the interest for 1 yr. on \$200 is \$12; on \$400 it is \$24; and on \$500 it is in line with 30, that is, the interest is \$30.



Axis of principals

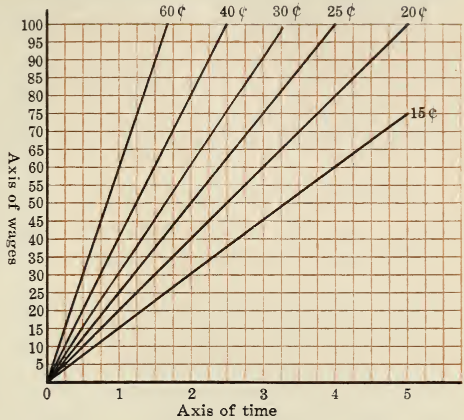
Such a small graph is given merely as an illustration, since the interest is easily found mentally. If the graph were constructed on a larger scale, it would be helpful as a check on computations. It is introduced here as a further step to the appreciation of the value of the graph.

In a similar way we can draw a graph showing the interest on \$100 at 6% for any periods of time reckoned in years. From this graph we see that the interest on \$100 for 1 yr. is \$6; for 2 yr., \$12; for 6 yr., \$36; and so on. By reversing the process we can see that \$6 is the interest for 1 yr., \$12 for 2 yr., and so on.



Axis of time

Wage Graph. A reduced portion of a graph used in the computation of wages is here shown. Since the wage for no time is 0, the graph originates in the intersection of the two axes, this being called the *origin*. At 15¢ per hour the graph must pass through a point above 1 and to the right of 15, or, as it is usually written, at the point (1, 15),



and hence we draw a line from the origin through (1, 15).

Likewise, at 20¢ per hour the graph passes from the origin through (1, 20), and so on.

To find the wages for $3\frac{1}{4}$ hr. at 25¢, we see that the vertical line from $3\frac{1}{4}$ cuts the 25-cent graph a little above 80¢.

In a large graph the result can be found to the nearest cent

Exercise 35. Wage Graphs

1. Draw a wage graph for 4 hr., arranged by quarter hours, the wages being 32¢ an hour.

2. From Ex. 1 find the wages for $2\frac{3}{4}$ hr. at 32¢.

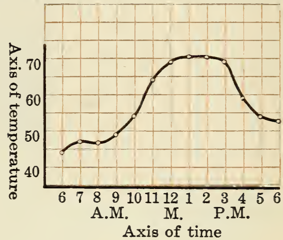
3. As in Ex. 1 draw a graph for wages of 36¢.

4. As in Ex. 1 draw a graph for wages of 48¢ and find the wages due a man for $3\frac{1}{2}$ hr. work.

Curves as Graphs. Graphs are not always straight like those on pages 53–57. For example, consider the graph of temperature here shown. The temperatures from 6 A.M. to 6 P.M. on a certain day, taken every hour, were as follows:

Hour	6	7	8	9	10	11	12	1	2	3	4	5	6
Degrees	45	48	$47\frac{1}{2}$	50	55	65	70	72	72	70	60	55	54

Representing the hours on the horizontal axis and the degrees on the vertical axis, marking the temperatures for the successive hours, and connecting the points, we have a curve which shows more clearly than the numbers alone the way in which the temperature varied during the day.



You may have seen a thermometer with an attachment for drawing such a graph continuously, day and night.

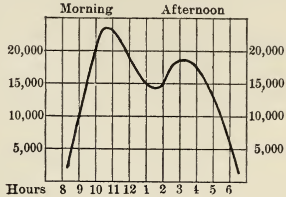
It is desirable that students should keep for a reasonable period records similar to the above and that they should draw the graph.

Exercise 36. Curves as Graphs

1. Draw a graph showing the changes in temperature on a day when the thermometer registered as follows: 6 A.M., 50°; 7 A.M., 55°; 8 A.M., 60°; 9 A.M., 62°; 10 A.M., 62°; 11 A.M., 70°; noon, 75°; 1 P.M., 72°; 2 P.M., 75°; 3 P.M., 65°; 4 P.M., 60°; 5 P.M., 58°; 6 P.M., 50°; 7 P.M., 48°; 8 P.M., 45°.

2. In Ex. 1 state from the graph the hour during which the temperature rose most rapidly; fell most rapidly; changed the least.

3. This graph shows the frequency of telephone calls in an important exchange near Wall Street, in New York City. Write a statement telling what you infer as to the time of the greatest amount of business, the cause for the depression between the two peaks, and the time when the rush of business is over.



4. The approximate distances of the horizon as seen from heights above the surface of the sea are as follows:

Height in feet	$88\frac{1}{2}$	200	400	500	550	1200	2212	3186
Distance in miles	10	15	$21\frac{1}{4}$	$23\frac{3}{4}$	25	37	50	60

Draw a graph, representing distances on the x-axis, each space being 5 mi., and representing heights on the y-axis, each space being 100 ft.

5. From the graph in Ex. 4 determine the height at which the sea horizon is distant 20 mi.; 30 mi.; 40 mi.

6. From the graph in Ex. 4 determine the distance that an aviator can see when 2000 ft. above the sea.

7. The number of years that a man may expect to live, as based on insurance statistics, is as follows:

Age	10	15	20	25	30	35	40	45	50
Expectation	48.7	45.5	42.2	38.8	35.3	31.8	28.2	24.5	20.9

Draw a graph, representing ages on the x-axis.

8. From the graph in Ex. 7 find the expectation of life at the age of 23; of 28; of 32; of 42; of 48.

9. The horse power (H. P.) of the engines of a certain type of steamship and the speed of the ship in miles per hour are shown by the following table:

Speed of the ship	5	10	12	15	16	20
Horse power	250	2000	3256	6750	8192	16,000

Draw a graph, representing H. P. on the x-axis and speed on the y-axis, each space on the x-axis being 1000 H. P.

10. From the graph in Ex. 9 find the H. P. of the engines of a steamship for a speed of 18 mi.

11. The following table gives the normal weight of boys and girls for certain ages, expressed in kilograms. Graph the data on the same paper, using the same scale.

Age	6	7	8	9	10	11	12	13	14	15	16	17	18
Boys	19.7	21.6	23.8	26.3	28.7	31.2	34.2	38.1	42.7	48.0	53.2	57.4	61.3
Girls	18.9	21.0	23.0	25.3	27.8	30.7	34.4	39.0	44.2	48.2	50.7	52.4	53.0

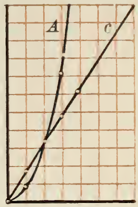
12. The following table shows the normal strength of right-hand grip for boys and girls of certain ages, expressed in kilograms. Graph the data on the same paper.

Age	6	7	8	9	10	11	12	13	14	15	16	17	18
Boys	9.2	10.7	12.4	14.3	16.5	18.9	21.2	24.4	28.4	33.4	39.4	44.7	49.3
Girls	8.4	9.9	11.2	12.8	14.7	16.5	18.9	21.8	24.8	27.0	28.7	29.6	29.8

13. Construct a graph for converting yards into rods, and rods into yards.

14. Construct a graph for converting Fahrenheit temperature into centigrade.

Graph of a Formula. Many formulas assume a new meaning by the study of graphs. For example, consider the formulas $c = 2\pi r$ and $A = \pi r^2$.



Giving r, in turn, the values 0, 1, 2, 3, ..., we easily prepare the following table:

If $r =$	0	1	2	3	4
then $c =$	0	6.3	12.6	18.9	25.1
and $A =$	0	3.1	12.6	28.3	50.3

Then the graphs of c and A are as shown in the margin.

The vertical scale in the graph shown above is taken as one fourth the horizontal scale, because otherwise the height of the figure would be inconvenient.

We learn from the graph that at first the circumference changes much more rapidly than the area changes; for example, as r increases from 0 in. to 1 in. the circumference increases from 0 in. to 6.3 in., while the area increases to only 3.1 sq. in.; but soon after this the area begins to increase with very great rapidity, while the circumference simply increases at the same rate as at the beginning.

Exercise 37. Graphs of Formulas

1. As above, draw the graphs of $c = 2\pi r$, representing the circumference of a sphere; $A = 4\pi r^2$, representing the surface; and $V = \frac{4}{3}\pi r^3$, representing the volume. What do the three graphs show as to the rate of increase of each?

2. Draw a graph of $V = \pi r^2 h$ for the fixed value $h = 10$; also for the fixed value $r = 2$. In which case does the volume of a cylinder change more rapidly: when h is fixed and r changes or when r is fixed and h changes?

Exercise 38. Review

1. Draw a graph of $a = \frac{1}{2}bh$ for the fixed value $b = 8$; for the fixed value $h = 6$.

2. Draw a graph for converting meters into feet.

3. Draw a graph for converting kilograms into pounds.

4. Draw a graph for converting gallons into liters.

5. The following table shows the number of miles of railway built in the United States between certain dates. Graph the data.

Years	1860–1870	1870–1880	1880–1890	1890–1900	1900–1910
Miles	22,000	40,000	66,000	35,000	46,000

6. In a certain year the average per capita cost of elementary schools in 62 cities was \$36.95. During the same year the per capita cost, expressed in dollars, in several cities, was as stated below. Graph the data by bars.

City	A	B	C	D	E	F	G	H	I
Per capita	28.00	30.00	32.14	32.71	33.19	35.04	37.19	37.91	47.64

Per capita cost means the cost per person.

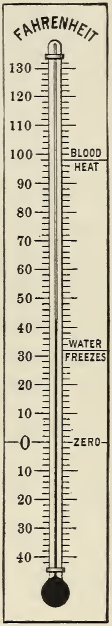
7. In a certain year the average per capita cost of high-school education in 54 cities was \$78.74. During the same year the per capita cost, expressed in dollars, in several cities, was as stated below. Graph the data by bars.

City	A	B	C	D	E	F	G	H
Per capita	48.00	66.49	68.11	71.39	79.04	91.00	108.00	150.00

8. Graph the noon temperatures of your city for five consecutive days.

IV NEGATIVE NUMBERS

Need for Negative Numbers. In the summer time, in our country, when we speak of the temperature we always mean the number of degrees above zero. But in the winter, in the northern part, the temperature often falls below zero.



We can indicate temperature below zero by writing, for example, "three degrees below zero," but we naturally use a kind of shorthand and write 3° instead of "three degrees." We have also a kind of shorthand for the expression "below zero"; that is, we commonly write $-3°$ to mean 3° below zero. This is usually read "minus 3°," or sometimes "negative 3°." It does not mean that 3° has been subtracted from anything; it simply means that 3° is to be considered below zero.

Teachers should call attention to the fact that we use *a* for altitude and area, *gr.* for grain and gross, and various other symbols for two or more different things. This never confuses us, because our common sense tells us what meaning is to be taken. Likewise we use the symbol $-$ and the word "minus" to mean both subtraction and a number below zero; no confusion comes from this, because our common sense tells us which meaning is to be taken in a given case.

The number -3 is called a *negative number*, and the number 3 is called a *positive number*. If we wish to emphasize the fact that 3 is positive, we may place the sign $+$ before it, thus, $+3$; but this is not often done. We therefore see that 3° and $-3°$ mean 3 degrees respectively above zero and below zero. The *numerical value* of a number is the value without reference to the sign, 3 and -3 having the same *numerical* value.

Examples of Negative Numbers. Not only do we use negative numbers to express temperature below zero, but we use them for many other purposes. For example:

Debt. If a man is worth \$1000, we may say that he has + \$1000; but if he has no money and is \$1000 in debt, we may say that he is worth – \$1000. *A debt may be expressed by a negative number.*

Latitude. If we call latitude north of the equator positive, we speak of latitude south of the equator as negative. *South latitude may be expressed by a negative number.*

Longitude. If we call longitude west of Greenwich positive, we speak of east longitude as negative. *East longitude may be expressed by a negative number.*

Of course if we should speak of south latitude as positive, we would speak of north latitude as negative, and similarly for longitude.

Pressure. If we call downward pressure, like a weight, positive, *we may speak of a balloon inflated with gas lighter than air as having negative weight.*

Altitude. If we speak of the altitude of a mountain as positive, *we may speak of depth below the sea level as negative altitude.*

Direction. If we take any direction as positive, we naturally speak of the opposite direction as negative. *Negative direction is the opposite of positive direction.*

Games. In some games there are negative scores; that is, we may have a score of -10, which means that we are 10 worse off than nothing. *A negative score in a game means a score below zero.*

We therefore see that negative numbers are just as real as positive numbers, for the temperature is just as real when the mercury in a thermometer is below zero as when it rises above zero.

Exercise 39. Negative Numbers

All work oral

1. If you say that Mr. A is worth $-$ \$500, what do you mean?

2. If you say that the latitude of Buenos Ayres is $-34° 36' 30''$, what do you mean?

3. If your geography tells you that the longitude of Berlin is $-13° 23' 43''$, what do you understand this to mean?

4. If a toy balloon pulls upward with a force of 1 oz., how may you describe the weight of the balloon?

5. Where is a point located with respect to sea level if its altitude is spoken of as -1500 ft.?

6. If three boys are pulling against three others in a tug of war, and we speak of a motion of the rope to the north as positive, how may we describe motion to the south?

7. In throwing bean bags some girls marked a number of squares on the floor. One of the squares was marked -5. If the first bean bag thrown fell in this square, what was the girl's score, and what did it mean?

8. If the temperature in Duluth was $-15°$ on a January day, how many degrees must it rise to be $-14°$? to be $-10°$? to be $0°$? to be $5°$?

9. If we call noon of to-day zero, what negative number represents 11 A.M. to-day?

10. If we speak of midnight as zero, what number represents 9 P.M. of the preceding day?

11. About how many feet above the floor is the ceiling of your schoolroom? We may, therefore, say that the ceiling is about how many feet below the floor?

The student should see that one answer requires a negative number.

Addition. Since we now see that negative numbers are just as real, in many cases, as positive numbers, we naturally expect that they can be added like other numbers. This is actually the case; for if a man is worth $-\$500$, that is, if he is \$500 in debt, it becomes very real if he gets $-\$200$ added to what he has, that is, if he gets \$200 more in debt. In other words, $-\$500+(-\$200)=-\$700$, and the man is \$700 in debt.

In order to avoid confusion we often use parentheses as above.

Therefore, *to add a negative number to a negative number, add as if the numbers were positive and then prefix the negative sign.*

Likewise, if a man is \$500 in debt, that is, if he is worth $-\$500$, and someone gives him \$400, he can pay part of his debt and owe only \$100. That is, $-\$500+\$400=-\$100$.

Similarly, if a man is worth $-\$500$ and someone gives him \$700, he can pay all his debt and have \$200 over. That is, $-\$500+\$700=\$200$.

Therefore, *to add a positive number to a negative number, consider their numerical values only, find their difference, and then prefix the sign of the numerically greater number.*

This principle may be further illustrated by answering the following questions:

1. If your score in a game is -5 and your next mark is -3, what is then your total score?

2. If your score in a game is -10 and you make 8, what is then your score?

3. If your score in a game is -6 and you make 10, what is then your score?

4. If the temperature in Maine is $-7°$ some morning and the temperature rises $15°$ during the forenoon, what is then the temperature?

Exercise 40. Addition

Examples 1 to 5, oral

1. To a flatiron weighing 6 lb. a boy ties a small balloon that weighs -1 lb. What is the total weight?

2. A man begins business with \$1000. During the first year he gains \$500. How much is he then worth?

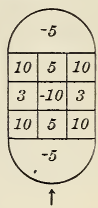
3. In Ex. 2 the man gains $-$\$700 the second year. How much is he then worth?

4. In Exs. 2 and 3 the man gains \$1200 the third year. How much is he then worth?

5. A man begins business with $-$\$500. What does this mean? The first year he gains \$900. How much is he then worth?

6. An airplane that can maintain a speed of 63.5 mi. an hour in still air is flying against a wind that retards it 10.2 mi. an hour. What distance does the airplane actually make in an hour?

That is, $63.5 + (-10.2)$ is equal to what number?



7. A game is played by throwing bean bags in the direction of the arrow. Suppose that the score stands -10, 3, 3, 5, -5, 3, 10, -10, -5, 10, 10, how much is the total score?

8. If this board without any weights at the ends just balances, and if I put 5 lb. at one end and 8 lb. at the other end, how much must I add to the 5 lb. to make it balance? Instead of adding to the 5 lb., how much might I add to the 8 lb.?

9. A boat that goes 16.8 mi. an hour in still water is going against a stream which retards it 4.2 mi. an hour. What distance does the boat actually make in an hour?

Subtraction. If the temperature is 60° in the morning and 80° at noon, the difference in temperature is 20°; that is, we must add 20° to 60° to make 80°.

But suppose that the temperature on a winter morning in St. Paul is $-4°$ and rises to 10° at noon, what is the difference in temperature? That is, what must we add to $-4°$ to make 10°?

An easy way to answer this is first to see what must be added to $-4°$ to bring the temperature up to 0°, and then to see what more must be added to bring it up to 10°.

Evidently we must add 14° to $-4°$ to make 10°; that is,

$$10-(-4)=10+4=14.$$

We therefore see that *the subtraction of a negative number is the same as the addition of the positive number having the same numerical value.*

Now consider what must be added to 10 to make -4. Evidently we must add -10 to make 0, and -4 more to make -4; that is,

$$-4-10=-4+(-10)=-14.$$

Therefore *the subtraction of a positive number is the same as the addition of the negative number having the same numerical value.*

It is not necessary to remember these principles, for all we need do in any case is to see what number must be added to the number which is subtracted to make the other number.

Before considering Exercise 41, it is well to answer the following questions, drawing a diagram to illustrate each:

What must be added to $-6°$ to make $-5°$? How much is $-5-(-6)$?

What must be added to $-8°$ to make 8°? How much is $8-(-8)$?

Exercise 41. Subtraction

Examples 1 to 21, oral

1. A man having only \$5 went to a store and paid for \$5 worth of goods. How much money had he left?

2. A man having only \$5 went to a store and bought \$8 worth of goods, paying what he could. How much had he left, including his debt? Explain the answer.

3. If the thermometer indicates 60° at 8 A.M. and 70° at noon, how much has the temperature risen?

4. If the temperature is 70° at 8 A.M. and 60° at noon, how much has it risen? What does the answer mean?

State the results of the following:

5. $8-4$.	**9.** $13-5$.	**13.** $-4-2$.
6. $4-8$.	**10.** $5-13$.	**14.** $-4-(-2)$.
7. $8-(-4)$.	**11.** $13-(-5)$.	**15.** $-2-4$.
8. $4-(-8)$.	**12.** $5-(-13)$.	**16.** $-2-(-4)$.

17. State in two ways the difference between -5 and 10; between 7 and -6; between 0 and -8.

18. What is the difference in longitude between New York, which has a longitude of 73°, and Berlin, which has a longitude of $-13°$, each to the nearest degree?

State the answer in two ways, depending on whether you measure from Berlin to New York or from New York to Berlin.

19. How much better off is a man who is worth \$5000 than one who is \$1000 in debt?

20. How much better off is a man who is worth exactly nothing than one who is \$2000 in debt?

21. How much better off is a man who is worth $-$\$500 than one who is worth $-$\$1500?

Perform the following additions and subtractions:

22. $250 - (-50)$.

23. $250 + (-50)$.

24. $50 - 250$.

25. $50 - (-250)$.

26. $50 + (-250)$.

27. $-50 + 250$.

28. $375 - 75$.

29. $375 - (-75)$.

30. $375 + (-75)$.

31. $75 - 375$.

32. $75 - (-375)$.

33. $75 + (-375)$.

34. A man having $750 incurs a debt of $1000. How much is his balance then?

35. A man having $1000 incurs a debt of $750. How much is his balance then?

36. A motorist starts from a place 75 mi. south of Denver and motors to a place 150 mi. north of Denver. How many miles has he motored? Draw a rough diagram to illustrate the problem.

In all such problems assume that he motors in a straight line.

37. A ship in $-22° 17'$ latitude sails to a place in $17° 26'$ latitude. Through how many degrees of latitude has it sailed? Draw a rough diagram to illustrate.

Performing the operations in the order indicated, find the value of each of the following:

38. $275 + 328 + (-142) + (-196) - 25 - (-2)$.

39. $962 - 327 - (-327) + (-140) - (-137)$.

40. $782 - (-782) + (-782) - 438 - (-296) + (-7)$.

41. An airplane which has a rate of 67.8 mi. an hour in still air is retarded by the wind 17.6 mi. in the first hour of a trip and 8.4 mi. in the second hour. How far does it go in the 2 hr.?

How much is $2 \times 67.8 + (-17.6) + (-8.4)$?

Multiplication. We can easily see by considering the following illustrations what multiplication means when negative numbers are involved:

1. If a boy saves \$3 a month, how much better off will he be 2 mo. hence, that is, + 2 mo. from now?

Evidently he will be 2 × \$3 better off, or \$6 better off.

2. If a boy saves \$3 a month, how much better off was he 2 mo. ago than he is now, that is, − 2 mo. from now?

Evidently he was not better off, but \$6 worse off; that is,

$$(-2) \times \$3 = -\$6.$$

3. If a boy wastes \$3 a month, that is, if he thus saves − \$3 a month, how much better off will he be 2 mo. hence as a result of this?

Evidently he will not be better off, but \$6 worse off; that is,

$$2 \times (-\$3) = -\$6.$$

4. If a boy wastes \$3 a month, that is, if he thus saves − \$3 a month, how much better off was he 2 mo. ago, that is, − 2 mo. from now?

Evidently he had wasted \$6 less, and hence was \$6 better off: that is,

$$(-2) \times (-\$3) = \$6.$$

Laws of Signs. From the above illustrations we find the following laws:

Plus × *plus* = *plus*, as in Ex. 1

Minus × *plus* = *minus*, as in Ex. 2

Plus × *minus* = *minus*, as in Ex. 3

Minus × *minus* = *plus*, as in Ex. 4

Or, stated more concisely,

In multiplication two like signs produce plus; two unlike signs produce minus.

Exercise 42. Multiplication

Examples 1 to 5, oral

1. If 8 men, each of whom benefits a city \$2 a day, move into the city, what is the total benefit per day? How much is 8×2?

2. If 8 men, each of whom benefits a city \$2 a day, move out of the city, how is the net income of the city affected? How much is $(-8) \times 2$?

3. If 8 men, each of whom is a charge upon a city of \$2 a day, move into the city, how is the net income of the city affected? How much is $8 \times (-2)$?

4. If 8 men, each of whom is a charge upon a city of \$2 a day, move out of the city, how is the net income of the city affected? How much is $(-8) \times (-2)$?

5. If you save 10¢ each week, how much better off will you be 3 wk. hence? How much better off or worse off were you 3 wk. ago, that is, -3 wk. hence?

Perform the following multiplications:

6. $27 \times (-36)$.

7. $(-42) \times 68$.

8. $39.8 \times (-2.9)$.

9. $(-75) \times (-126)$.

10. $(-8.4) \times (-63.8)$.

11. $(-12\frac{7}{8}) \times (-23\frac{3}{4})$.

12. In building schoolhouses a formula is often used for the height of the chalk trough in a schoolroom. This formula is $h = 25 + \frac{3}{2}(g - 4)$, where h is the height of the chalk trough in inches and g the number of the grade. Find the height for Grade VIII, that is, when $g = 8$.

13. In Ex. 12 find the height for Grade IV; that is, find h when $g = 4$. Find the height when $g = 3$.

How much is $g - 4$ in each case? How much is $\frac{3}{2}$ of 0? $\frac{3}{2}$ of -1?

Division. Since division is the opposite of multiplication, we can obtain the laws of signs very easily. That is,

Because $2 \times 3 = 6$, we see that $6 \div 3 = 2$.
Because $(-2) \times (-3) = 6$, we see that $6 \div (-3) = -2$.
Because $(-2) \times 3 = -6$, we see that $(-6) \div 3 = -2$.
Because $2 \times (-3) = -6$, we see that $(-6) \div (-3) = 2$.

We may state these laws as follows:

Plus $\div$ *plus* $=$ *plus*, as in $6 \div 3 = 2$
Plus $\div$ *minus* $=$ *minus*, as in $6 \div (-3) = -2$
Minus $\div$ *plus* $=$ *minus*, as in $(-6) \div 3 = -2$
Minus $\div$ *minus* $=$ *plus*, as in $(-6) \div (-3) = 2$

Or, stated more concisely,

In division two like signs produce plus; two unlike signs produce minus.

Exercise 43. Division

Examples 1 to 13, oral

Perform the following divisions:

1. $8 \div 2$.	**5.** $6 \div 2$.	**9.** $2 \div 1$.
2. $8 \div (-2)$.	**6.** $6 \div (-2)$.	**10.** $2 \div (-1)$.
3. $(-8) \div 2$.	**7.** $(-6) \div 2$.	**11.** $(-2) \div 1$.
4. $(-8) \div (-2)$.	**8.** $(-6) \div (-2)$.	**12.** $(-2) \div (-1)$.

13. If the temperature has risen $-16°$ in 2 hr., that is, if it has fallen $16°$, find the average rise per hour.

Perform the following divisions:

14. $1728 \div (-12)$.	**18.** $(-49.02) \div (-3.8)$.
15. $(-14.4) \div 1.2$.	**19.** $(-3.456) \div (-1.44)$.
16. $(-28.8) \div (-1.2)$.	**20.** $(-17{,}028) \div 13.2$.
17. $13{,}020 \div (-12.4)$.	**21.** $223.11 \div (-6.7)$.

Exercise 44. Review

1. A manufacturer bought six bales of cotton, supposed to weigh 500 lb. per bale. The real weights were 492 lb., 512 lb., 488 lb., 508 lb., 514 lb., and 490 lb. How many pounds did he actually buy?

Instead of adding these numbers, it is much easier to do as is often done in similar cases in business; that is, to write the numbers as $500 - 8$, and so on, as here shown. The result will be 6×500, together with the sum of -8, 12, -12, 8, 14, and -10.

$$\begin{array}{|l|}\hline 500 - 8 \\ 500 + 12 \\ 500 - 12 \\ 500 + 8 \\ 500 + 14 \\ 500 - 10 \\ \hline \end{array}$$

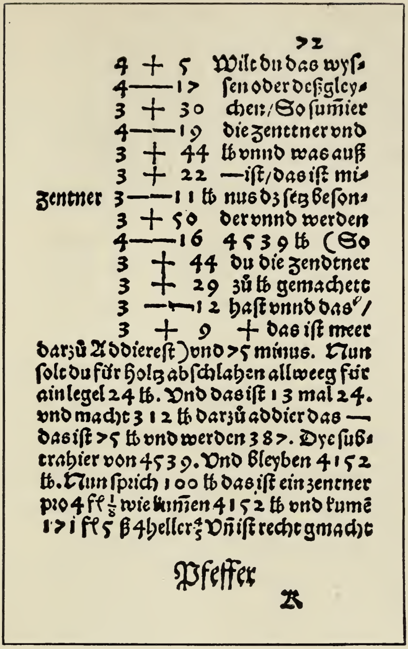
2. The level of the water in a standpipe rises 3 ft. 9 in., then falls 4 ft. 8 in., rises again 15 in., falls 2 ft. 4 in., rises 3 ft. 2 in., falls 1 ft. 9 in., and finally rises 15 in. How much higher or lower is it than at first?

3. The highest point in North America, Mount McKinley, Alaska, is 20,300 ft. above the sea. The lowest point, -276 ft., is in Death Valley, California. Find the difference.

4. The highest mountain in the world is Mt. Everest, 29,002 ft. The deepest natural depression except the ocean is the Dead Sea, 1290 ft. below sea level. What is the difference between these extremes?

5. An elevator in a store made the following trips in a given time: It started from the ground floor, ascended to the fifth floor above the ground floor, descended to the first floor below the ground floor, ascended to the sixth floor, descended to the second floor below the ground floor, ascended to the ground floor. Draw a graph showing its progress. Indicate the negative directions.

6. The temperature in a certain city was 24° yesterday at 3 P.M., and it grew colder at an average rate of 2.5° an hour. What was the temperature at 10 P.M.?



72
4 + 5 Wiltu das wyſ-
4 — 17 ſen oder desgleych-
3 + 30 chen/So ſumier
4 — 19 die zentner vnd
3 + 44 lb vnnd was auß
3 + 22 —iſt/das iſt mi-
zentner 3 — 11 lb nus dz ſetz beſon-
3 + 50 der vnnd werden
4 — 16 4539 lb (So
3 + 44 du die zendtner
3 + 29 zů lb gemachet
3 — 12 haſt vnnd das +/
3 + 9 + das iſt meer
darzů Addiereſt) vnd 75 minus. Nun
ſolt du für holtz abſchlahen allweeg für
ain legel 24 lb. Vnd das iſt 13 mal 24.
vnd macht 312 lb darzů addier das —
das iſt 75 lb vnd werden 387. Dye ſub-
trahier von 4539. Vnd bleyben 4152
lb. Nun ſprich 100 lb das iſt ein zentner
pro 4 fl $\frac{1}{5}$ wie kummen 4152 lb vnd kume
171 fl 5 ß 4 heller $\frac{4}{5}$ Vnd iſt recht gmacht

Pfeffer

R

FIRST PLUS AND MINUS SIGNS PRINTED

This is a page from the first printed book in which the symbols + and — appeared. The book was written by Johann Widman and was printed in Germany in 1489, three years before America was discovered. It was one of the first books on mathematics printed in any German state. The symbols were used to indicate that certain packages contained so many pounds more or less than a given number of hundredweights. The case is similar to that of Ex. 1 on the opposite page, where it is easier to add $500 - 8$, $500 + 12$, $500 - 12$, $500 + 8$, $500 + 14$, and $500 - 10$ than to add 492, 512, 488, 508, 514, and 490. It is interesting to know that in actual business life to-day the same process is used as is found in this old book printed more than four centuries ago. The plus signs and minus signs were not used in an algebraic equation until some years later, and it was still later before the full significance of negative numbers was as well understood by many prominent mathematicians as you understand it to-day. It is also interesting to see how the forms of the numerals have changed in the last four hundred years

7. The longitude of Chicago is $87^\circ\ 36'\ 42''$ and that of Paris, France, is $-2^\circ\ 20'\ 15''$. What is the longitude of a place halfway between the two cities?

8. A bullet leaves the muzzle of a rifle with a velocity of 1200 ft. per second. It is fired backward from a moving car, and the actual velocity of the bullet is 1180 ft. per second. At what rate per hour is the car moving?

9. A man has a bank account of \$589 and a house worth \$2850. There is a mortgage of \$1475 on his house and he has other obligations amounting to \$1130. How much is he worth?

10. The greatest known depth in the ocean is in the Pacific, 32,088 ft. The highest point on the land is Mt. Everest, 29,002 ft. What is the difference in elevation?

11. In playing a certain game a boy first makes a score of 45; he is then set back 20; he then scores 60, and is next set back 85. What is his final score?

12. An explorer starts at $3^\circ\ 20'\ 18''$ south latitude. He goes north $7^\circ\ 5'\ 30''$, then south $4^\circ\ 20'$, then north $1^\circ\ 10'\ 20''$, then south $2^\circ\ 4'\ 18''$. What is then his latitude?

13. The highest and lowest temperatures ever reported from certain cities up to a recent date were as follows:

Place	San Francisco	New Haven	Savannah	Chicago	Duluth
Highest	101°	100°	105°	103°	99°
Lowest	29°	– 14°	8°	– 23°	– 41°

Compute for each city the extremes of temperature, that is, the difference between the highest and the lowest temperature. Compute the per cent of variation in each city compared with the variation in San Francisco.

V. ALGEBRAIC OPERATIONS

Addition. We have already learned how to add such expressions as $2x$, $3x$, and $7x$. Since we have also learned how to add negative numbers, we can easily see how to add $2x$, $-3x$, and $7x$. To be sure that the principle is understood, a few simple problems are given below.

Exercise 45. Addition

All work oral

1. If you have 10¢, earn 5¢, and spend 7¢, how much have you left? How much is $10+5-7$? How much is $10x+5x-7x$?

2. If you have 4 doz. oranges, sell 2 doz., and then buy 3 doz., how many dozen will you have? How much is $4-2+3$? How much is $4d-2d+3d$? $4x-2x+3x$?

State the results of the following additions:

3.	4.	5.	6.	7.
3 doz.	3 books	$3x$	$3M$	$3n$
– 5 doz.	– 5 books	$-5x$	$-5M$	$-5n$
7 doz.	7 books	$7x$	$7M$	$7n$

8. Add $5x$, $2x$, $-3x$, and x.

Remember that x is the same as $1x$.

State the results of the following additions:

9.	10.	11.	12.	13.
$5a$	$9x$	$7m$	$5ab$	$2(a+b)$
$-2a$	$4x$	$-2m$	$6ab$	$-2(a+b)$
$3a$	x	$8m$	$-2ab$	$7(a+b)$
$6a$	$-7x$	$-5m$	$-4ab$	$-5(a+b)$
$-a$	$-5x$	$-3m$	$-5ab$	$3(a+b)$

Further Work in Addition. We add algebraic expressions much as we add feet and inches. For example:

2 ft. 3 in.	2 lb. 3 oz.	$2x + 3y$
4 ft. 5 in.	4 lb. 5 oz.	$4x + 5y$
6 ft. 8 in.	6 lb. 8 oz.	$6x + 8y$

We may also have some negative numbers, thus:

5 ft. + 9 in.	15 lb. − 4 oz.	$12x + 7y$
3 ft. − 6 in.	12 lb. − 5 oz.	$3x - 9y$
8 ft. + 3 in.	27 lb. − 9 oz.	$9x - 2y$

Exercise 46. Addition

Perform the following additions:

1.
7 ft. 2 in.
3 ft. 9 in.

2.
7 yd. 2 in.
3 yd. 9 in.

3.
$7x + 2y$
$3x + 9y$

4.
$7 \cdot 4 + 2 \cdot 5$
$3 \cdot 4 + 9 \cdot 5$

5.
$5a + 2b$
$7a - 3b$

6.
$8x - 4y$
$3x - 5y$

7.
$14a - 7b$
$3a + 7b$

8.
$15b + 9c$
$-15b - 9c$

9.
2 yd. + 1 ft. + 6 in.
7 yd. + 1 ft. + 4 in.

10.
$2x + y + 6z$
$7x + y + 4z$

11.
$15p - 4q + 3r$
$-15p + 4q - 3r$

12.
$17a - 6b + 7c$
$5a + b - 3c$

Find the sum of each of the following:

13. $-11\,a,\ 3\,b,\ c,\ -20\,a,\ -8\,b$, and $-2\,c$.

14. $15\,d,\ -3\,e,\ 4f,\ -7\,d,\ 3\,e,\ -7f,\ -4\,e$, and $12f$.

15. $4.5\,a-3.4\,b+7.3\,c,\ \ -7.5\,a+5.6\,b-4.5\,c$.

16. $5\,h+4\,t+u,\ 7\,h+3\,t+u$, and $-3\,h+t+5\,u$.

17. $14\,x+7\,y-3\,z,\ 7\,x-3\,y-4\,z$, and $8\,x+2\,y-9\,z$.

Perform the following additions:

18.

$$\begin{array}{r} 14\text{ mi.}\quad 2\text{ rd.}\ 3\text{ ft.} \\ \underline{7\text{ mi.}\ 18\text{ rd.}\ 9\text{ ft.}} \end{array}$$

19.

$$\begin{array}{r} 4\text{ mi.} + 3\text{ rd.} + 2\text{ yd.} \\ \underline{3\text{ mi.} - 5\text{ rd.} + 1\text{ yd.}} \end{array}$$

20.

$$\begin{array}{r} 17\text{ km.} + 4\text{ m.} + 7\text{ cm.} \\ \underline{37\text{ km.} + 9\text{ m.} - 3\text{ cm.}} \end{array}$$

21.

$$\begin{array}{r} 15\text{ T.} + 18\text{ lb.} - 3\text{ oz.} \\ \underline{7\text{ T.} - \ \ 5\text{ lb.} - 4\text{ oz.}} \end{array}$$

Exs. 18–21 are inserted, of course, merely to show the analogy of algebra and arithmetic. They are not otherwise practical.

22.

$$\begin{array}{r} -3\,(a+b)-4\,(c+d) \\ \underline{7\,(a+b)+5\,(c+d)} \end{array}$$

23.

$$\begin{array}{r} 5\,(3\,x+2\,y)-3\,(4\,a+2\,b) \\ -7\,(3\,x+2\,y)+5\,(4\,a+2\,b) \\ \underline{8\,(3\,x+2\,y)-9\,(4\,a+2\,b)} \end{array}$$

24.

$$\begin{array}{rrrrr} 3\,abc & +4\,a^2 & +7\,b & & \\ & -5\,a^2 & -3\,b & +8\,c & \\ -7\,abc & & +9\,b & -4\,c & +3 \\ \underline{9\,abc} & \underline{+8\,a^2} & \underline{} & \underline{-2\,c} & \underline{-5} \end{array}$$

25.

$$\begin{array}{r} 13\,(x-y)-4\,(z+w) \\ \underline{5\,(x-y)+8\,(z+w)} \end{array}$$

26.

$$\begin{array}{r} 3\,(e-f)+4\,(x-y) \\ -7\,(e-f)-8\,(x-y) \\ \underline{5\,(e-f)+9\,(x-y)} \end{array}$$

27.

$$\begin{array}{rrrr} 4\,t^2 & -3\,tu & +\ \ 5\,u & +7 \\ & 7\,tu & -\ \ 8\,u & -9 \\ 8\,t^2 & -8\,tu & +11\,u & +4 \\ \underline{} & \underline{7\,tu} & \underline{+\ \ 4\,u} & \underline{-8} \end{array}$$

Relation to Numbers. We come to understand better our ordinary work with numbers if we see its relation to the work in algebra. For example, we may write $7t+5$ for 75 if we understand that t stands for 10, and we may add several such numbers, as here shown.

$7t+\ 5$	75
$3t+\ 7$	37
$9t+\ 2$	92
$19t+14$	204

We naturally wonder why the results are not more alike. The reason is that 10 units make 1 ten, and so, when we have 14 units, we think of 1 ten + 4 units, adding the 1 ten to the tens. We shall get exactly the same result in the addition at the left if we let $t=10$, for we shall have $19\times 10+14=190+14=204$.

The teacher should see that the whole subject of "carrying" in addition is here explained to the students.

Exercise 47. Addition

Add the following columns and show how the results in each example can be made to appear alike:

1.

$7t+8$	78
$4t+9$	49
$6t+1$	61

2.

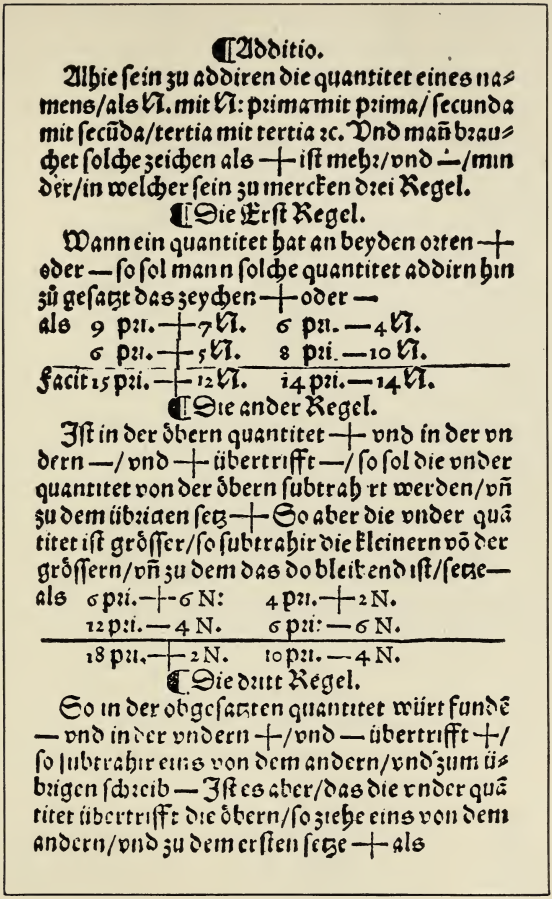
$9t+4$	94
$6t+3$	63
$5t+0$	50
$7t+7$	77

3.

$5h+7t+4$	574
$3h+2t+9$	329
$8h\qquad+6$	806

4.

$8h+9t+0$	890
$6t+7$	67
$5h\qquad+9$	509
$8h+7t+3$	873



Additio.

Alhie sein zu addiren die quantitet eines na-
mens/als N. mit N: prima mit prima/ secunda
mit secūda/tertia mit tertia ꝛc. Vnd mañ brau-
chet solche zeichen als + ist mehr/vnd — /min
der/in welcher sein zu mercken drei Regel.

Die Erst Regel.

Wann ein quantitet hat an beyden orten +
oder — so sol mann solche quantitet addirn hin
zu gesatzt das zeychen + oder —

als 9 pri. + 7 N. 6 pri. — 4 N.
6 pri. + 5 N. 8 pri. — 10 N.
Facit 15 pri. + 12 N. 14 pri. — 14 N.

Die ander Regel.

Ist in der obern quantitet + vnd in der vn
dern — / vnd + übertrifft — / so sol die vnder
quantitet von der obern subtrahirt werden/vñ
zu dem übrigen setz + So aber die vnder quā
titet ist grösser/so subtrahir die kleinern vō der
grössern/vñ zu dem das do bleibend ist/setze —

als 6 pri. + 6 N: 4 pri. + 2 N.
12 pri. — 4 N. 6 pri: — 6 N.
18 pri. + 2 N. 10 pri. — 4 N.

Die dritt Regel.

So in der obgesatzten quantitet würt fundē
— vnd in der vndern +/vnd — übertrifft +/
so subtrahir eins von dem andern/vnd zum ü-
brigen schreib — Ist es aber/das die vnder quā
titet übertrifft die obern/so ziehe eins von dem
andern/vnd zu dem ersten setze + als

EARLY USE OF PLUS AND MINUS

This illustration is from one of the oldest printed books on algebra, and shows how students added $9x + 7$ and $6x + 5$ four hundred years ago. The book was written in Vienna in 1518 by Henricus Grammateus. This was before mathematicians had begun to use letters like a, b, x, and y to represent numbers

Subtraction. Suppose that we wish to take 3 ft. 1 in. from 9 ft. 8 in., 3 lb. 1 oz. from 9 lb. 8 oz., and 31 from 98. The work will evidently be as follows:

9 ft. 8 in.	9 lb. 8 oz.	98
3 ft. 1 in.	3 lb. 1 oz.	31
6 ft. 7 in.	6 lb. 7 oz.	67

We may, if we wish, have some negative numbers in such subtractions. Consider the following:

12 ft. + 9 in.	12 lb. − 8 oz.	$12t+8$
6 ft. − 4 in.	6 lb. − 4 oz.	$6t-4$
6 ft. + 13 in.	6 lb. − 4 oz.	$6t+12$

Exercise 48. Subtraction

Perform the following subtractions:

1.
9 ft. 7 in.
2 ft. 4 in.

2.
9 yd. 7 in.
2 yd. 4 in.

3.
$$\begin{array}{r} 9x+7y \\ \underline{2x+4y} \end{array}$$

4.
$$\begin{array}{r} 8x+5y \\ \underline{2x-3y} \end{array}$$

5.
$$\begin{array}{r} 18a+4b \\ \underline{9a-6b} \end{array}$$

6.
$$\begin{array}{r} 25m-12n \\ \underline{12m-25n} \end{array}$$

7.
$$\begin{array}{r} 2a+3b+9c \\ \underline{a+2b+7c} \end{array}$$

8.
$$\begin{array}{r} 8x+7y-5z \\ \underline{7x-2y-3z} \end{array}$$

9.
$$\begin{array}{l} 9m+4n \\ \underline{6m-8n-s} \end{array}$$

Relation to Numbers. Letting h stand for 100 and t for 10, we may write $7h+8t+4$ for 784. Let us take two numbers written this way and subtract, thus:

$7h+8t+4$	784
$3h+2t+1$	321
$4h+6t+3$	463

From this we see how much subtraction in arithmetic resembles subtraction in algebra. If we have a case in which the lower digit exceeds the upper one, there seems at first to be a difference, as in the following problem:

$7h+2t+4$	724
$3h+8t+1$	381
$4h-6t+3$	343

There really is not any difference, for if $h=100$ and $t=10$, then $4h-6t+3$ is the same as $400-60+3$, or 343.

Exercise 49. Subtraction

Perform the following subtractions:

1.

$9t+3$	93
$6t+2$	62

2.

$8t+1$	81
$4t+7$	47

3.

$7h+6t+4$	764
$3h+t$	310

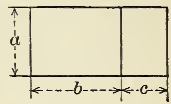
4.

$9h+7t+3$	973
$2h+9t+6$	296

Multiplication. If we wish to multiply 7 ft. 2 in. by 4, we multiply 7 ft. by 4 and then multiply 2 in. by 4, adding the products; and if we wish to multiply $7x+2y$ by 4, we proceed in the same way. Consider the following:

7 ft. 2 in.	$7x+2y$	$7t+2$	72
4	4	4	4
28 ft. 8 in.	$28x+8y$	$28t+8$	288

Consider also the case of the product $a(b+c)$. Here we may think of the base of a rectangle as being $b+c$ and the height as being a. The area of the whole rectangle is evidently $a(b+c)$, that of the left-hand part is ab, and that of the right-hand part is ac, so that $a(b+c) = ab + ac$.



In the case of $a(a+b)$ we should have $a^2 + ab$.

Exercise 50. Multiplication

Perform the following multiplications:

1.	**4.**	**7.**	**10.**
9 ft. 3 in.	$8x+7y$	$x+y$	$x+2a$
3	5	a	a
2.	**5.**	**8.**	**11.**
$9f+3i$	$6a+8b$	$x+y$	$x+3a$
3	2	x	a
3.	**6.**	**9.**	**12.**
$9x+3y$	$9m+6n$	$x+y$	$4x+1$
3	5	y	b

Relation to Numbers. Letting $h = 100$ and $t = 10$, we may write $6h + 2t + 3$ for 623, and multiply by 3 thus:

$$\begin{array}{r} 6h + 2t + 3 \\ 3 \\ \hline 18h + 6t + 9 \end{array} \qquad \begin{array}{r} 623 \\ 3 \\ \hline 1869 \end{array}$$

Suppose, however, that we have 689 to be multiplied by 3. We should then have the following:

$$\begin{array}{r} 6h + \ \ 8t + \ \ 9 \\ 3 \\ \hline 18h + 24t + 27 \end{array} \qquad \begin{array}{r} 689 \\ 3 \\ \hline 2067 \end{array}$$

But since we know that in this case $h = 100$ and $t = 10$, we may write $18h + 24t + 27$ as $20h + 6t + 7$.

Exercise 51. Multiplication

Perform the following multiplications:

1.
$$\begin{array}{r} 7t + 1 \\ 9 \\ \hline \end{array} \qquad \begin{array}{r} 71 \\ 9 \\ \hline \end{array}$$

2.
$$\begin{array}{r} 9t + 3 \\ 2 \\ \hline \end{array} \qquad \begin{array}{r} 93 \\ 2 \\ \hline \end{array}$$

3.
$$\begin{array}{r} 8t + 2 \\ 3 \\ \hline \end{array} \qquad \begin{array}{r} 82 \\ 3 \\ \hline \end{array}$$

4.
$$\begin{array}{r} 8h + 2t + 3 \\ 2 \\ \hline \end{array} \qquad \begin{array}{r} 823 \\ 2 \\ \hline \end{array}$$

5.
$$\begin{array}{r} 8h + t + 2 \\ 4 \\ \hline \end{array} \qquad \begin{array}{r} 812 \\ 4 \\ \hline \end{array}$$

6.
$$\begin{array}{r} 9h + 4t + 8 \\ 3 \\ \hline \end{array} \qquad \begin{array}{r} 948 \\ 3 \\ \hline \end{array}$$

Negative Multiplier. Since we have learned the law of signs in multiplication (page 71), we can multiply when negative numbers appear. That is,

$$-2 \times (-3) = 6, \qquad -a(b - c) = -ab + ac,$$
$$-2 \times (-a) = 2a, \qquad -a(a - b) = -a^2 + ab.$$

For example, suppose that we are required to multiply $2x - 7y$ by $-4x$. Arranging the work as here shown and multiplying, we have as a result $-8x^2 + 28xy$.

$$\boxed{\begin{array}{r} 2x - 7y \\ -4x \\ \hline -8x^2 + 28xy \end{array}}$$

We may begin at either the right or the left to multiply, the latter often having some advantages over the plan commonly followed in arithmetic.

Exercise 52. Multiplication

Perform the following multiplications:

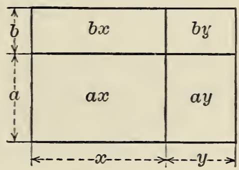
1. $\begin{array}{r} a+b \\ a \\ \hline \end{array}$	**5.** $\begin{array}{r} x-y \\ -z \\ \hline \end{array}$	**9.** $\begin{array}{r} 2x+3y \\ 3x \\ \hline \end{array}$	**13.** $\begin{array}{r} 9a+7b \\ 6b \\ \hline \end{array}$
2. $\begin{array}{r} a-b \\ a \\ \hline \end{array}$	**6.** $\begin{array}{r} x+y \\ -y \\ \hline \end{array}$	**10.** $\begin{array}{r} 3x-4y \\ 5y \\ \hline \end{array}$	**14.** $\begin{array}{r} 8a-9b \\ -4b \\ \hline \end{array}$
3. $\begin{array}{r} a-b \\ -a \\ \hline \end{array}$	**7.** $\begin{array}{r} x+a \\ -x \\ \hline \end{array}$	**11.** $\begin{array}{r} 4x-7y \\ -3y \\ \hline \end{array}$	**15.** $\begin{array}{r} 7m-6n \\ -9m \\ \hline \end{array}$
4. $\begin{array}{r} a+b \\ -a \\ \hline \end{array}$	**8.** $\begin{array}{r} x-a \\ -x \\ \hline \end{array}$	**12.** $\begin{array}{r} 5x-8y \\ -4x \\ \hline \end{array}$	**16.** $\begin{array}{r} 8m-7n \\ -4n \\ \hline \end{array}$

Multiplier of Two Terms. In an expression like $3a^2 + 4b$ we speak of the two *terms*, the terms in this case being $3a^2$ and $4b$. If a multiplier has two terms, we may multiply as in arithmetic when the multiplier has two figures, thus:

$$\begin{array}{r}
3t+2 \\
\underline{2t+1} \\
\overline{3t+2} \\
6t^2+4t \\
\hline
6t^2+7t+2
\end{array}
\qquad
\begin{array}{r}
30+2 \\
\underline{20+1} \\
\overline{30+2} \\
600+40 \\
\hline
600+70+2
\end{array}
\qquad
\begin{array}{r}
32 \\
\underline{21} \\
\overline{32} \\
64 \\
\hline
672
\end{array}$$

We may show by a diagram how to multiply $x + y$ by $a + b$, thus:

$$\begin{array}{l}
x+y \\
a+b \\
\hline
ax+ay+bx+by
\end{array}$$



In this case we begin at the left to multiply. We need not write the partial products under one another, because we cannot add the terms in the same way as at the top of the page.

Exercise 53. Multiplication

Perform the following multiplications:

1.	3.	5.	7.
$\begin{array}{r} 2x+3 \\ \underline{3x+2} \end{array}$	$\begin{array}{r} 5a+2 \\ \underline{3a-7} \end{array}$	$\begin{array}{r} 12p+7 \\ \underline{3p-7} \end{array}$	$\begin{array}{r} 7m+n \\ \underline{7m-n} \end{array}$

2.	4.	6.	8.
$\begin{array}{r} 4x+7 \\ \underline{2x-1} \end{array}$	$\begin{array}{r} 6m+4 \\ \underline{3m-2} \end{array}$	$\begin{array}{r} a+2 \\ \underline{a-2} \end{array}$	$\begin{array}{r} 8a+b \\ \underline{8a-b} \end{array}$

A Special Product. Algebra teaches us many facts which we can use practically in arithmetic. For example, consider the product of $x + y$ and $x + y$, that is, the square of $x + y$. Beginning at the left in multiplying, we have the following:

$$\begin{array}{r} x + y \\ \underline{x + y} \\ x^2 + xy \\ \underline{ xy + y^2} \\ x^2 + 2\,xy + y^2 \end{array} \qquad \begin{array}{c|c|c|} \cline{2-3} y & xy & y^2 \\ \cline{2-3} x & x^2 & xy \\ \cline{2-3} \multicolumn{1}{c}{} & \multicolumn{1}{c}{x} & \multicolumn{1}{c}{y} \end{array}$$

This is easily seen from the diagram, where the area of a square that is $x + y$ on a side is $x^2 + xy + xy + y^2$, or $x^2 + 2\,xy + y^2$.

That is, $$(x + y)^2 = x^2 + 2\,xy + y^2.$$

We may now use this in arithmetic, as in finding the square of 25, thus:

$$\begin{aligned} 25^2 = (20 + 5)^2 &= 20^2 + 2 \times 20 \times 5 + 5^2 \\ &= 400 + 200 + 25 \\ &= 625. \end{aligned}$$

This is easier than to multiply in the usual way. In the case of 87^2 we first write 80^2, which is 6400; then $2 \times 80 \times 7$, which is 160×7, or 1120; and finally 7^2, which is 49. This is somewhat easier than the ordinary method of multiplying 87 by 87.

$$\boxed{\begin{array}{r} 6400 \\ 1120 \\ \underline{49} \\ 7569 \end{array}}$$

In the case of such simple numbers as 13, 31, 51, and 81 it is much easier to use this plan. For example,

$$13^2 = 100 + 60 + 9 = 169,$$
$$31^2 = 900 + 60 + 1 = 961,$$
$$51^2 = 2500 + 100 + 1 = 2601.$$

Another Special Product. Another product which will prove useful in arithmetic is the one found by multiplying $x+y$ by $x-y$. Here we see that

$$(x+y)(x-y)=x^2-y^2.$$

$$\begin{array}{l} x + y \\ \underline{x - y} \\ x^2 + xy \\ \underline{\quad\; - xy - y^2} \\ x^2 \qquad\quad - y^2 \end{array}$$

This enables us to do some interesting multiplication in our heads. For example, it would seem rather surprising if a boy gave the product of 37 and 43 without writing down the work, if we did not know the short method which algebra teaches. But it is easy to see that

$$\begin{aligned} 43 \times 37 &= (40+3)(40-3) \\ &= 1600 - 9 \\ &= 1591. \end{aligned}$$

Similarly, we have the following products:

$$102 \times 98 = (100+2)(100-2) = 10{,}000 - 4 = 9996.$$

$$305 \times 295 = (300+5)(300-5) = 90{,}000 - 25 = 89{,}975.$$

Exercise 54. Special Products

Examples 1 to 7, oral

Find the products of the following:

1. $(a+1)^2$.	**8.** 21^2.	**15.** 44×36.
2. $(x+2)^2$.	**9.** 32^2.	**16.** 53×47.
3. $(m+n)^2$.	**10.** 45^2.	**17.** 64×56.
4. $(p+7)^2$.	**11.** 57^2.	**18.** $(m+n)(m-n)$.
5. $(m+6)^2$.	**12.** 62^2.	**19.** $(2a+1)(2a-1)$.
6. $(n+5)^2$.	**13.** 38^2.	**20.** $(5a+7)(5a-7)$.
7. $(5+1)^2$.	**14.** 75^2.	**21.** $(7x+y)(7x-y)$.

Exercise 55. Review of Multiplication

Examples 1 to 8, oral

Multiply as indicated:

1. $(p+x)(p+x)$.

2. $(p+x)(p-x)$.

3. $(x+p)(x+p)$.

4. $(x+p)(x-p)$.

5. $(m+2)(m-2)$.

6. $(a+m)(a-m)$.

7. $(B+b)(B+b)$.

8. $(B+b)(B-b)$.

9. $(a+3b)(a+4b)$.

10. $(4a+b)(3a+b)$.

11. $(7+\frac{2}{3})(3+\frac{3}{4})$.

12. $(3x-7)(2x-3)$.

13. $(5x+6y)(5x-6y)$.

14. $(7x+2y)(3x-7y)$.

15. $(5a+3b)(7a-8b)$.

16. $(ab+1)(ab+1)$.

17. $(ab+1)(ab-1)$.

18. $(1-ab)(1+ab)$.

19. $(x-2y)(x-3y)$.

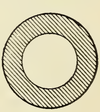
20. $(2x-3y)(3x-2y)$.

Write the results of the following:

21. 23^2.	**25.** 61^2.	**29.** 61×59.	**33.** 101×99.
22. 42^2.	**26.** 63^2.	**30.** 63×57.	**34.** 107×93.
23. 43^2.	**27.** 72^2.	**31.** 72×68.	**35.** 202×198.
24. 54^2.	**28.** 82^2.	**32.** 73×67.	**36.** 401×399.

37. In $A = \frac{1}{2}h(B+b)$, the formula for the area of a trapezoid, multiply $B+b$ by $\frac{1}{2}h$ and then write the formula so that it shall contain no parentheses.

38. One formula for the area of the metal in the cross section of a pipe is $A = \pi(R+r)(R-r)$, where R is the radius of the outer circle and r is the radius of the inner circle. Multiply $R+r$ by $R-r$ and multiply the product by π, expressing the formula so that it shall contain no parentheses.



In Exs. 37 and 38 it is much easier to compute with parentheses.

Division. If we wish to divide 9 ft. 3 in. by 3, we simply divide 9 ft. by 3 and then divide 3 in. by 3, adding the quotients; and if we wish to divide $9x + 3y$ by 3, we proceed in the same way. Consider the following cases:

3) 9 ft. 3 in.	$3\overline{)9x + 3y}$	$3\overline{)9t + 3}$	$3\overline{)93}$
3 ft. 1 in.	$3x + y$	$3t + 1$	31

We see that division in algebra is very much like division in arithmetic.

Because $\quad a(a + b) = a^2 + ab,$

we see that $\quad (a^2 + ab) \div a = a + b.$

Exercise 56. Division

Perform the following divisions:

1. 4) 12 lb. 8 oz.
2. $4\overline{)12x + 8y}$
3. 5) 15 ft. 10 in.
4. $5\overline{)15a + 10b}$
5. 6) 24 yd. 18 in.
6. $6\overline{)24m + 18}$
7. 8) 48 mi. 72 ft.
8. $8\overline{)48pq + 72}$
9. $9\overline{)81t + 9}$
10. $9\overline{)819}$
11. $12\overline{)144x + 48y}$
12. $15\overline{)225m^2 + 90}$
13. $a\overline{)a^2 + 2a}$
14. $a\overline{)a^2 + 4ab}$
15. $b\overline{)2b + b^2}$
16. $3\overline{)6h + 9t + 3}$
17. $3\overline{)693}$
18. $4\overline{)8h + 4t + 8}$
19. $4\overline{)848}$
20. $2a\overline{)2a + 4a^2}$
21. $3x\overline{)9x^2 + 3x}$
22. $5ab\overline{)5ab + 25a^2b^2}$
23. $25m\overline{)250m^2 + 25m}$
24. $2\frac{1}{2}x\overline{)5x^2 + 10x}$

Negative Divisor. Since we have learned the law of signs in division (page 73), we can divide when negative numbers appear. That is,

$$6 \div (-2) = -3, \qquad (-ab + ac) \div a = -b + c,$$
$$(-6) \div 2 = -3, \qquad (-ab + ac) \div (-a) = b - c,$$
$$(-6) \div (-2) = 3, \qquad (-ab - ac) \div (-a) = b + c.$$

For example, suppose that we are to divide $4a^2 - 16a$ by $-2a$. Arranging the work as here shown, we have as the result $-2a + 8$.

$$-2a\,\overline{)\,4a^2 - 16a}$$
$$\quad\;\; -2a + 8$$

Exercise 57. Division

Perform the following divisions:

1. $ax^2 + ay^2$ by a.
2. $amn - ax$ by a.
3. $amn - ax$ by $-a$.
4. $abc - abd$ by ab.
5. $abc - abd$ by $-ab$.
6. $wxy + xyz$ by xy.
7. $-abc + bcd$ by $-c$.
8. $-abc - bcd$ by $-bc$.
9. $x^2y + xy^2$ by x.
10. $xyz^2 - x^2yz$ by $-z$.
11. $xyz^2 - x^2yz$ by xy.
12. $xyz^2 + x^2yz$ by xyz.
13. $ax - ay$ by a.
14. $ax^2 - ay$ by $-a$.
15. $a^2mn - amn^2$ by $-a$.
16. $a^2mn - amn^2$ by amn.
17. $p^2qr + 4p^2q$ by p^2q.
18. $-16x^2y + 24y^2$ by $-8y$.

19. Writing the formula for the area of a trapezoid as $A = \frac{1}{2}Bh + \frac{1}{2}bh$, divide each member of the equation by $\frac{1}{2}h$.

20. Show that, if $t = 10$, the division of $72t + 8$ by 4 is substantially the same as the division of 728 by 4.

21. The division of what numbers is suggested by the division of $8h + 4t + 6$ by 2? Divide in both cases.

1. 2. 3. 4. 5. 6. 7. 8. 9. 10. 11. 12.
ℛ. ʒ. ℭ. ʒʒ. ß. ʒℭ. bß. ʒʒʒ. ℭℭ. ʒß. cß. ʒʒℭ.
13. 14. 15.
dß. ʒbß. ℭß. &c.

The firſt caracter written thus ℛ, doeth ſignifie any number before which it is written, to be the firſt number giuen, taken, or imagined;and is called *radix*, or roote,for that all the other caracters haue their originall or of-ſpring of it. The ſecond written thus ʒ, is called *zenſe* or ſquare, and doth ſignifie any number before which it is written, to be the product of the firſt multiplication of the roote by it ſelfe: that is of the roote two times taken and multiplied. The third written thus ℭ, is called *cube*, and doth ſignifie the number following the ſame to be the product of the ſecond multiplication of the roote,three times taked and multiplied, that is of the ʒ multiplied by the ℛ. The fourth is called *zenſezenſe*, and doth ſignifie the number following the ſame,to be the product of the third multiplication,that is the product of the ℛ foure times taken and multiplied. The fift is called *ſurſolide*, and doth ſignifie the number following the ſame, to be the product of the fourth multiplication. The ſixt is called *zenſecube*, and doth ſignifie the number following the ſame,to be the product of the fift multiplication. The ſeuenth is called *bſurſolide*, or ſecond ſurſolide, and doeth ſignifie the number following the ſame,to be the product of the ſixt multiplication. The eight is called *zenſezenſezenſe*, & doth ſignifie the number following the ſame,to be the product of the ſeuenth multiplication. The ninth is called *cubecube*, and doth ſignifie, the number following the ſame,to be the product of the eight multiplication. The tenth is called *zenſeſurſolide*, & doth ſignifie the number following the ſame, to be the product &c. The eleuenth, is called *cſurſolide*,or third ſurſolide. The twelfth is called *zenſezenſecube*. The thirteenth is called *dſurſolide*, or fourth ſurſolide. You may proceede further at your pleaſure, if

DIFFICULTIES OF ALGEBRA IN EARLY DAYS

Algebra is much easier than it was when the above page was printed. The page is from a book by Thomas Masterson, printed in London in 1592. Perhaps the student may care to read it and see how hard it was to begin the study of algebra a hundred years after Columbus sailed to America

Exercise 58. Miscellaneous Problems in Division

Perform the following divisions:

1. $a^2 \div a$.
2. $4\,a^2 \div 2$.
3. $6\,a^2 \div 3\,a$.
4. $9\,a^2 \div 3\,a^2$.
5. $18\,a^2 \div 9\,a$.
6. $axy \div x$.
7. $axy \div xy$.
8. $ax^2y \div axy$.
9. $25\,a^2 \div 2.5$.
10. $(-x^2y) \div (-x)$.
11. $(-x^2y) \div (-y)$.
12. $36\,x^2 \div (-3.6)$.
13. $(-39\,p^2) \div (-13\,p)$.
14. $(-48\,n^2) \div (-n^2)$.
15. $2.88\,x^2y \div 12\,x^2$.

Considering a fraction as an expression of division, perform the following divisions:

16. $\dfrac{75\,a^2b + 15\,b^2}{5\,b}$.
17. $\dfrac{32\,m^2n - 24\,n^2}{-8\,n}$.
18. $\dfrac{-81\,x^2 - 216\,x}{-9\,x}$.
19. $\dfrac{-48\,m^2 - 144\,m}{-24\,m}$.
20. $\dfrac{\frac{1}{2}\,h\,(B + b)}{\frac{1}{2}\,h}$.
21. $\dfrac{\frac{1}{2}\,hb + \frac{1}{2}\,hB}{\frac{1}{2}\,h}$.

Find the value of x in each of the following equations:

22. $2\,x = 28$.
23. $3\,x = 39\,a$.
24. $5\,ax = 75\,a$.
25. $5\,ax = 75\,a^2$.
26. $5\,ax = -75\,a^2b$.
27. $-3\,ax = 75\,a^2$.
28. $5\,a^2x = 75\,a^2b$.
29. $-pqrx = -p^2q^2r^2$.
30. $-37\,px = -74\,p^2q$.
31. $ax = a^2b + ab^2$.

32. Express $5 \div 25$ as a fraction, and also $a \div a^2$ as a fraction, each in its lowest terms.

33. Find the value of x in the equation $a^2x = a$.

34. Find the value of x in the equation $25\,a^2x = 5\,a$.

Exercise 59. Review

Perform the following additions:

1. $2b + 3b$.
3. $5a^2 + a^2$.
5. $2\frac{1}{2}a + \frac{3}{4}a$.
2. $3h + 4h$.
4. $m^2 + m^2$.
6. $7t + 3t$.

Simplify the following formulas:

7. $s = 2\pi r^2 + 2\pi rh$.
8. $a = p + prt$.

In Ex. 7 notice that we have to add $r \times 2\pi r$ to $h \times 2\pi r$. If we have to add rx to hx, the result is $(r + h)x$.

9. A shipper of apples kept track of the number of barrels left over or received by placing a + before the number, and of the number of barrels shipped by placing a − before the number. His record was $+46 + 30 - 20 - 16 - 20 + 60 - 40 + 10 - 30$. How many barrels were left?

10. In the formula $c = 2\pi r$, substitute πd for c. Then simplify the equation as much as you can and tell what the result means.

11. In the formula $a = p + prt$, substitute i for prt and tell what the result means.

12. In the formula $h = \frac{1}{33000}plan$, h represents the horse power of an engine, p the pressure of the steam in pounds per square inch, l the length of the piston rod in inches, a the area of the piston head in square inches, and n the number of strokes per minute. If $p = 120$, $l = 20$, $a = 132$, and $n = 60$, what is the horse power of the engine?

13. The formula $A = \sqrt{s(s-a)(s-b)(s-c)}$ gives the area of a triangle, where a, b, c are the sides of the triangle, and $s = \frac{1}{2}(a + b + c)$. Find the area of a triangle whose sides are 6 in., 8 in., 10 in.

When we speak of a, b, and c as the sides we mean the number of units of length, as is always understood in such cases.

Express in the form of equations the statements made in Exs. 14–21:

14. A certain number is greater by 8 than a second number d.

15. The sum of two numbers is 28, and the smaller is n.

16. The difference between two numbers is d, and the greater is g.

17. Of three consecutive numbers, the sum of $\frac{1}{2}$ the first and $\frac{1}{5}$ of the second is 9 less than the third.

18. One fifth of the sum of two consecutive numbers is three times their difference.

19. One fourth of a number which is 1 less than a certain number is 10 less than the latter number.

20. One eighth of a number which is 3 more than a certain number is equal to this certain number.

21. If the square of a certain number is decreased by 51, the result is equal to the number multiplied by 3 less than itself.

22. If $2n$ represents a certain even number, represent the next two odd numbers; the two preceding even numbers.

23. How can you represent your age n years ago? n years hence?

24. The length of the longest night in the year in certain latitudes is shown in the following table:

Latitude	25°	35°	50°	60°
Length	13 hr. 34 min.	14 hr. 22 min.	16 hr. 9 min.	18 hr. 30 min.

Draw the graph and find approximately the length of the longest night in your latitude.

VI. FURTHER USES OF ALGEBRA

Problems of Simple Machines. In books on mechanics, in the encyclopedias, and in journals relating to the shop, formulas are often found. We shall now give a few which you may possibly meet in school or which you may need soon after leaving school.

It is not essential that the student should understand the mechanical principle involved in each problem. All that is necessary at this time is that he should know that the problem is practical and that he should be able to handle the formula readily.

Exercise 60. Simple Machines

1. A boy reads that the formula for the number of revolutions per minute of a wheel in a machine is given by the formula $n = \frac{7}{22}\, s/d$, where n is the number of revolutions, d is the diameter of the wheel in inches, and s is the speed in inches of a point on the circumference. Find the value of n when $s = 250$ and $d = 1.7$.

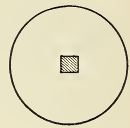
2. A book on shop mechanics states that the formula for the length of an open belt connecting two wheels on a machine is $L = \frac{13}{8}(D + d) + 2\,C$, where D is the diameter of the large wheel, d is the diameter of the small wheel, and C is the distance between the centers, all these measurements being in inches. Find the value of L when $D = 38$, $d = 18$, and $C = 72$.

The formula is only approximately correct, but it is often found in manuals.

3. In connecting two wheels by a belt it is often necessary to know the difference between the two circumferences. This is given by the formula $D = 2\,\pi(R - r)$. Suppose that D and r are known, find R.

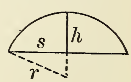
4. In a magazine on automobiles a boy reads that the formula for the horse power of a gasoline engine is $h = D^2N/2.5$, in which D represents the diameter of each cylinder in inches, and N the number of cylinders. If $D = 3$ and $N = 6$, what is the horse power of the engine?

5. In each of a lot of circular discs of metal there is to be punched a square hole as shown in the figure. A book on mechanics states that if from a circle of radius r there is cut a square of side s the area that remains is given by the formula $A = \pi r^2 - s^2$. If the radius of each disc is 3 in. and the side of the square is $\frac{1}{4}$ in., what is the area of each face of the remaining portion of the disc after the hole is punched?



6. The cross section of a metal bar is an equilateral triangle each side of which is s. The area of the cross section is given by the formula $A = \frac{1}{4} s^2\sqrt{3}$. If $\sqrt{3} = 1.73$, find the area of the cross section when $s = 6.4$.

7. The foreman of a shop reads that the safe load (l) in pounds that can be hoisted by a certain kind of rope c inches in circumference is given by the formula $l = 100c^2$. How many pounds can he safely allow for a rope the circumference of which is 3.1 in.?



8. A contractor wishes to build over an engine a circular arch of height h and span $2s$. It is necessary to find the radius of the circle so that he may make his pattern. He knows that $r = (s^2 + h^2)/2h$. Find the radius, given $h = 3$, $s = 5$.

9. In Ex. 8, what is the value of r when $h = s$?

10. From the formula $l = ar^2$, write the formula for a in terms of the other letters.

Problems in Business. As we have already found, algebra is often very helpful in business. A few further illustrations of such uses of algebra will now be given.

Exercise 61. Problems in Business

1. A trade price list gives the cost in dollars per running foot of sewer pipe of diameter d inches, as follows:

$$c = 0.004\, d^2 + 0.14.$$

Find the cost of 500 ft. of 18-inch pipe.

2. From the formula of Ex. 1, find the cost of a mile of 20-inch pipe.

3. From the formula of Ex. 1, find the cost of 150 yd. of 9-inch pipe.

4. If D and d are the external and internal diameters of a cast-iron pipe in inches, the weight of the pipe in pounds per foot is given by the formula $W = 2.45\,(D^2 - d^2)$. Find the weight of 250 ft. of such pipe if the external diameter is 16 in. and the internal diameter is $15\frac{1}{4}$ in.

5. The profits of a merchant this year are $17\frac{1}{2}\%$ more than they were last year. This year the profits are \$20,915. How much were the profits last year?

6. There is a formula for n, the net price of goods sold at a reduction of R per cent and r per cent from l, the list price. The formula is $n = l\,(1 - R\%)\,(1 - r\%)$. Find the net price when $l = 800$, $R = 10$, and $r = 8$.

7. A manufacturer charges m dollars to the cost of raw material, h dollars to overhead (that is, overhead charges in his business), w dollars to wages, and e dollars to other expenses. Write a formula for the profit at $r\%$ on the total expenses.

Problems relating to Mixtures. In many factories and laboratories it is necessary to mix certain substances, and the proportions are often found most simply by algebra.

1. How much water must be added to 1 qt. of a 25% solution of alcohol (that is, when 25% is pure alcohol and the rest water) to reduce it to a 20% solution?

Let $x =$ the number of quarts to be added.

Then $1 + x =$ the total number of quarts.

Then 20% of $(1 + x) =$ the number of quarts of alcohol,

or $\frac{1}{5}(1 + x) =$ the number of quarts of alcohol.

But from the problem we know that 25% of the quart is pure alcohol; that is, that $\frac{1}{4}$ qt. is pure alcohol.

Therefore $\frac{1}{5}(1 + x) = \frac{1}{4}$.

Multiplying by 5, $1 + x = 1\frac{1}{4}$.

Subtracting 1, $x = \frac{1}{4}$.

Therefore $\frac{1}{4}$ qt. of water must be added.

CHECK. If we add $\frac{1}{4}$ qt. of water to a mixture of $\frac{3}{4}$ qt. of water and $\frac{1}{4}$ qt. of alcohol, we have $\frac{5}{4}$ qt. of mixture of which $\frac{1}{4}$ qt., or $\frac{1}{5}$, is alcohol.

2. How much vinegar must be added to 1 gal. of a 50% solution of standard vinegar to make it a 75% solution?

We see that we have 4 qt. to begin with, of which half is pure vinegar; that is, we have 2 qt. of pure vinegar to begin with.

Let $x =$ the number of quarts to be added.

Then $4 + x =$ the total number of quarts.

Then $\frac{3}{4}(4 + x) =$ the number of quarts of vinegar.

But from the problem we know that this must be $2 + x$, because we had 2 qt. at first and have added x quarts.

Therefore $\frac{3}{4}(4 + x) = 2 + x$.

Multiplying by 4, $3(4 + x) = 8 + 4x$.

Simplifying, $12 + 3x = 8 + 4x$.

Subtracting $3x$ and 8, $4 = x$.

Therefore 4 qt. of standard vinegar must be added.

Exercise 62. Problems relating to Mixtures

Examples 1 to 5, oral

1. In the equation $2\%(1+x)=40\%$, what is the first step in the solution? the second step?

2. If we add x quarts to 1 qt. and take 20% of the sum, how shall we indicate the result?

3. If a certain ore yields 3% of its weight in pure metal, how many pounds does it yield to the ton (2000 lb.)?

4. If a certain ore yields 80 lb. of pure metal to the ton, what per cent does it yield?

5. In Ex. 4, suppose that the ore yields 40 lb. of pure metal to the ton.

6. How much silver must be added to 12 oz. of an alloy which contains 10% of pure silver in order to make the alloy contain 20% of silver?

7. How much water must be added to a gallon of a 30% solution of alcohol to reduce it to a 20% solution?

8. How much water must be added to 60 gal. of alcohol 95% pure to reduce it to an 80% solution?

9. How many pounds of water must be added to 10 lb. of a 5% solution of salt to reduce it to a 3% solution?

10. How much water must be added to a pint of a 6% solution of a certain drug to reduce it to a 3% solution?

11. How much water must be added to 40 gal. of an acid 85% pure to reduce it to a 60% solution?

12. How much acid must be added to 60 gal. of the acid which is 60% pure to make it an 80% solution?

13. How much alcohol must be added to 10 gal. of a mixture that contains 25% pure alcohol in order that the mixture shall then contain 50% pure alcohol?

Exercise 63. Problems with Numbers

1. Six more than twice a certain number is equal to 12. Find the number.

2. Four times a certain number is equal to 35 diminished by the number. Find the number.

3. Five times a certain number is equal to 44 diminished by six times the number. Find the number.

4. Nine times a certain number is equal to 39 diminished by four times the number. Find the number.

5. Twelve times a certain number is equal to 45 diminished by three times the number. Find the number.

6. Fourteen times a certain number is diminished by 2, the result being 26. Find the number.

7. Fifteen times a certain number is diminished by 7, the result being 38. Find the number.

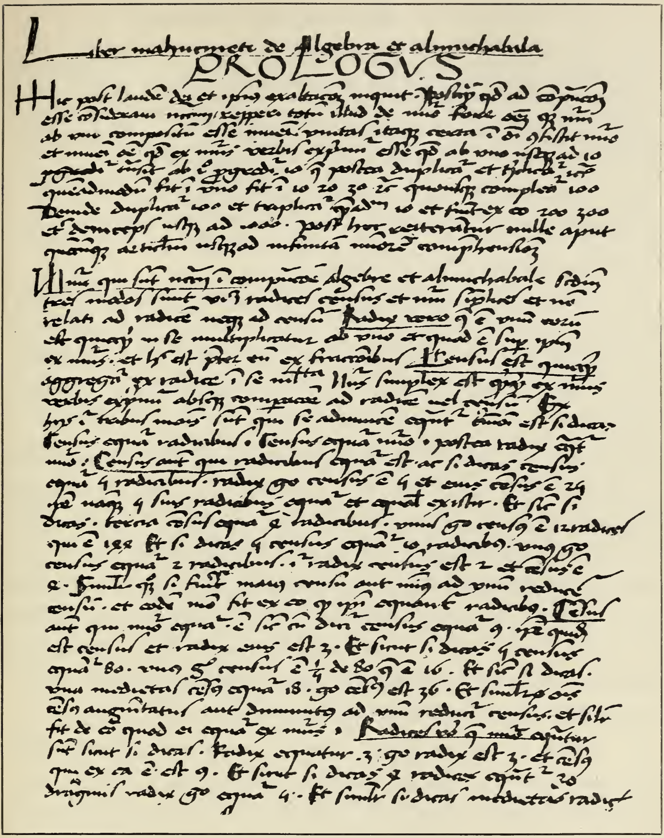
8. Twenty times a certain number is diminished by 7, the result being 21 more than twice the number. Find the number.

9. Thirty times a certain number is increased by 6, the result being 29 more than seven times the number. Find the number.

10. Forty-two times a certain number is increased by 18, the result being 92 more than five times the number. Find the number.

11. Sixty-nine times a certain number is increased by three times the number, the result being 134 more than five times the number. Find the number.

12. From $8\frac{1}{2}$ times a certain number there is subtracted 5, the result being 8 more than $5\frac{1}{4}$ times the number. Find the number.


Liber mahumeti de Algebra et almuchabala

PROLOGUS

OLD MANUSCRIPT ON ALGEBRA

The word "algebra" was first used, so far as we know, by a man named Mohammed ibn Musa, who lived at Bagdad at the time described in the Arabian Nights Tales, about 800 A.D. The subject was already old, but he gave it the name. His book was translated from Arabic into Latin about seven hundred years ago, and this illustration is from the first page of a manuscript of the translation copied in 1456. Algebra was not easy to read in those days, as you can see by glancing at this reproduction of the manuscript

Exercise 64. Miscellaneous Problems

1. The average monthly temperatures for one year in a place in the frigid zone were as follows: $-30°$, $-38°$, $-23°$, $0°$, $12°$, $21°$, $38°$, $27°$, $15°$, $4°$, $-14°$, $-24°$. What was the average of the monthly temperatures?

2. Air is 14.38 times as heavy as hydrogen gas. Hydrogen gas is what per cent as heavy as air? Air is what per cent as heavy as hydrogen gas?

3. A man had a square pond 45 ft. on a side, which was frozen to a depth of 9 in. The ice was cut to within 4 ft. of the shore and sold to a dealer at 13¢ per hundred pounds. Ice weighing $57\frac{1}{2}$ lb. per cubic foot, find to the nearest dollar how much money was received for the ice.

4. Water in freezing expands 10% of its volume. In a plant which manufactures artificial ice the molds are $24'' \times 36'' \times 12''$. How many cubic inches of water are needed to form enough ice to just fill the mold?

5. A wholesaler sold to a retailer at a profit of 15% on the cost of the goods, and the retailer sold to the consumer at a profit of 30% on the price he paid the wholesaler. Draw a graph for finding the cost to the wholesaler when the selling price to the consumer is known.

6. Think of any positive number, double it, add 4, add three times the original number, subtract 1, subtract five times the original number. The result is 3, no matter what the original number was. Explain why this is the case.

7. Make up a puzzle problem similar to that in Ex. 6.

8. The cost of housekeeping per week for a family which consists of n persons is sometimes given by the formula $c = 7.5 + 2.5\,n$. From this formula determine the number of persons in a family whose weekly expenses are \$22.50.

9. Write the formula for division in which D represents the dividend, Q the quotient, d the divisor, and R the remainder. Check the formula by substituting numbers.

10. A physician who has 10 oz. of an 8% solution of a certain medicine wishes to reduce it to a 5% solution. How much water must he add?

11. Given that $s = 10,320$, $v = 22$, $t = 40$, find the value of f in the formula $s = vt + \frac{1}{2}ft^2$.

Express Exs. 12–15 in algebraic symbolism:

12. The cost of n apples if x apples cost c cents.

13. The sum of any three consecutive numbers.

14. The excess of x over n.

15. The difference between two numbers is d, and the greater is 42.

16. The horse power (H.P.) of a pump which delivers G gallons of water per minute through a pipe d inches in diameter is given by the formula H.P. $= 4.2\ G^3/d^4$. Find the horse power necessary to deliver 8 gal. of water per minute through a 4-inch pipe.

17. The approximate height above sea level at which perpetual snow is found in different latitudes is given in the following table:

Latitude	0°	10°	20°	30°	40°	50°	60°
Height in feet	15,250	14,750	13,500	11,500	9000	6350	3800

Graph the data of the above table.

18. The sum of the angles of any triangle is 180°. The largest angle of a certain triangle is twice the first and three times the second angle. How many degrees in each angle?

Exercise 65. Review

1. In the formula $S = 16t^2 + vt$, given that $t = 10$ and $v = 5$, find the value of S.

2. If 40 lb. of sea water contains 2 lb. of salt, how much fresh water must be added to make a solution such that 15 lb. of it shall contain $\frac{1}{2}$ lb. of salt?

3. Given that $C = \frac{5}{9}(F - 32)$, find the value of C when $F = 32$; when $F = 77$.

4. Given that $S = (rl - a)(r - l)$, find the value of a in terms of the other letters.

5. One side of a rectangular frame building is l feet long and h feet high, and contains two windows each of which is a feet long and b feet high. What is the area of the wall surface of that side?

6. How could you represent algebraically any fraction whose denominator is greater by 5 than its numerator?

7. A recent census report divides the gainful occupations of the United States into the classes stated below. The figures indicate the per cent of the total number of workers who are engaged in the particular occupations. Represent these data by a circular pictogram.

Agriculture, forestry, and animal husbandry	33.2%
Extraction of minerals	2.5%
Manufacture and mechanical industries	27.9%
Transportation	6.9%
Trade	9.5%
Public service (not elsewhere classified)	1.2%
Professional service	4.4%
Domestic and personal service	9.9%
Clerical occupations	4.5%

PART II. ARITHMETIC

I. ARITHMETIC OF TRADE

Ordering Goods. Most of us are called upon at times to order goods by mail, and we should be able to do so in a businesslike way. The following is a model order for goods:

MARSHALL AND WILDER
BOOKSELLERS

Grand Rapids, Mich., Oct. 19, *19*20

Ginn and Company, Publishers
2301 Prairie Ave., Chicago

Dear Sirs,

Please send at once by express,

135 Wentworth-Smith: Complete Arithmetic,
60 Parker: Methods of Teaching in High Schools.

Yours truly,
Marshall and Wilder

Every business house of any pretensions uses letterheads giving its name, the nature of its business, and its address.

Exercise 1. Ordering Goods

1. Write an order to M. Smith & Co. of New York City for a half dozen tennis balls.

2. Write an order to L. E. Shields & Co. of Dallas, Texas, for a dozen fountain pens of a special kind.

3. Write an order to some firm for 3 doz. boxes of crayons of a certain kind.

Invoices and Bills. An *invoice* or a *bill* is an itemized statement of goods bought.

A physician's or a dentist's statement of services rendered, or a lumber company's statement of the amount of lumber used in building a house and the charges for the same, is called a *bill*; but a statement of a quantity of muslin bought or sold by a wholesale dry-goods merchant in the course of trade is called either a *bill* or an *invoice*. Mathematically a bill is the same as an invoice.

Model Invoice. The following model shows a common form of bills and invoices. A business house uses bills with a printed heading which states its name. An invoice should state the date and place of the sale, the names of the purchaser and the seller, the quantities and prices of the goods sold, and the terms.

The following is a form of the invoice that Ginn and Company would send in reply to the order on page 107:

Oct. 20, 1920

Marshall & Wilder
Grand Rapids, Mich.

Bought of GINN AND COMPANY
Educational Publishers
2301 Prairie Avenue, Chicago

Terms: Cash $\frac{1}{6}$

135 W.–S.: Complete Arithmetic,	60¢	\$81.00
60 Parker: Methods of Teaching in High Schools	1.50	90.00
		\$171.00
	Less $\frac{1}{6}$	28.50
		\$142.50

It is customary to state per cents like $16\frac{2}{3}\%$, $12\frac{1}{2}\%$, 20%, and 25% in the common-fraction form, as above. In this case $16\frac{2}{3}\%$ is stated as $\frac{1}{6}$. If the discount had been $12\frac{1}{2}\%$, it would have been stated as $\frac{1}{8}$.

Retailer's Bill. The following is a retailer's receipted bill:

Columbus, Ohio, Sept. 12, 1919

Mr. J. R. Adams
416 Water St., City

Bought of

M. W. CHAPMAN COAL CO.
267 First Street

Terms: Cash

Aug.	4	2 loads lump coal 6280–2180 6220–2120 8200# 7.00	28	70		
Aug.	14	2 loads lump coal 6640–2140 6220–2120 8600# 7.00	30	10		
					58	80
		RECEIVED PAYMENT M. W. CHAPMAN COAL CO. Per B. M. C.				

The figures at the left of the dash indicate the gross weight of the coal and wagon, and those at the right the weight of the wagon.

Exercise 2. Invoices and Bills

Date, fill, foot, and receipt the following, inserting in each the name and address of some purchaser and of some firm:

1. Bill for dry goods: $4\frac{1}{2}$ yd. percale @ 15¢; $12\frac{1}{2}$ yd. embroidery @ 25¢; 4 yd. lawn @ 22¢; $\frac{1}{4}$ yd. satin @ \$1.60.

2. Bill for groceries: 2 lb. coffee @ 35¢; 1 lb. tea, 50¢; 3 lb. lard @ 22¢; $2\frac{1}{2}$ lb. butter @ 38¢; 3 doz. eggs @ 34¢.

3. Bill for athletic goods: 3 bats @ 65¢; 2 bats @ 75¢; 4 baseballs @ 85¢; 2 baseballs @ \$1; 1 catcher's mitt, \$2.50; 1 catcher's mask, \$2.25.

Accounts. The following is a page from an account book:

1919		Receipts			1919		Payments		
Oct.	1	On hand	2	36	Oct.	4	Magazines		30
	5	From mother		40		9	Street-car tickets		50
	9	Errands		20		18	Stationery		40
	17	Birthday gift, Mary	1	50		22	Stamps		35
	30	From father		25		26	Candy		15
						31	Balance on hand	3	01
			4	71				4	71

Every student should be trained to keep simple accounts. The accounts should be of actual money belonging to the student, if this is possible. It is better to have a brief account which is a real one than to have a long account filled with imaginary transactions. Ask the students to bring to school certain bills which have been used in business transactions. The teacher should see that the work in accounts is made as practical as is easily possible.

The dollar sign and the decimal point are commonly omitted when the ruling of the account book shows clearly, as in the above illustration, which column is for dollars and which is for cents.

In the following form the account is credited with money received (Cr.), and debited with money paid out (Dr.).

1920		George Williams	Cr.		Dr.	
March	1	To cash on hand	4	30		
	7	By ticket for show				75
	8	By necktie				50
	14	By street-car tickets			1	00
	21	To allowance	2	50		
	22	By laundry				80
	28	By charity				30
	28	By magazines				40
	31	By balance			3	05
			6	80	6	80

Exercise 3. Accounts

Rule a sheet of paper, or use an account book properly ruled, and write the following accounts, balancing each:

1. May 1, on hand, \$3.14. Receipts: May 4, allowance from father, \$1; May 17, sale of garden produce, 85¢; May 24, sale of baseball mitt, 60¢. Payments: May 7, baseball, 50¢; shoes, \$2.50; May 18, ticket to circus, 50¢.

2. June 1, on hand, \$4.37. Receipts: June 3, cutting grass, 40¢; June 12, sale of flowers, \$1.75; June 17, sale of lettuce, 20¢; sale of eggs, 90¢; June 23, errands, 20¢. Payments: June 4, chicken feed, \$1.50; June 9, shoes, \$3; June 17, concert ticket, 50¢; June 19, ice cream, 10¢.

3. W. R. Weber, a farmer, trades with T. H. Gere, a grocer. The following purchases were made by Mr. Weber: Mar. 2, 2 lb. coffee @ 30¢; Mar. 4, 25 lb. sugar @ 6¢; bread, 25¢; Mar. 9, canned goods, \$2.50; Mar. 17, tea, 60¢; Mar. 23, 1 bbl. flour, \$9.60; Mar. 28, soap, 50¢; salt, 25¢; bread, 25¢. Mr. Gere bought the following from Mr. Weber: Mar. 18, 20 bu. potatoes @ 80¢; Mar. 21, 7 doz. eggs @ 35¢; Mar. 27, 9 doz. eggs @ 34¢. Make out a statement that would be rendered on Apr. 1 by T. H. Gere to W. R. Weber showing the balance due on that day.

Write the following accounts, inserting dates and items, and balance each:

4. On hand, \$2.37. Receipts: 30¢, \$2.25, 45¢, 15¢, \$1.12. Payments: \$1.45, 25¢, 45¢, \$1.20, 85¢, 65¢.

5. On hand, \$3.18. Receipts: 40¢, \$2.10, 30¢, 85¢. Payments: \$2.50, 10¢, 45¢, 30¢, \$1.10.

6. On hand, \$1.32. Receipts: 75¢, \$2.40, 35¢, 10¢, 25¢. Payments: 75¢, 25¢, \$2.50, 40¢, 10¢.

Profit and Loss. The business man must frequently ask himself the question, "Does it pay?" Arithmetic is often used to determine the answer to this question.

Gains and losses are frequently estimated at some rate per cent of the cost, of the money invested, or, in modern business, of the selling price.

The difference between the selling price and the cost price including all expenses is called the *gross profit.*

Illustrative Problem. A boy buys a baseball glove for \$1.75 and sells it for \$2. What per cent of the cost does he gain?

$$\begin{array}{r} \$2.00 \\ 1.75 \\ \hline \$0.25 \text{ gain} \end{array}$$

$$\$0.25 \div \$1.75 = 0.14\tfrac{2}{7}$$

Since 25¢ is gained on a cost of \$1.75, we need to find what per cent 25¢ is of \$1.75. Here we have the product, \$0.25, and one factor, \$1.75, to find the other factor; and $\$0.25 \div \$1.75 = 0.14\frac{2}{7}$. Hence he gains $14\frac{2}{7}\%$.

From this illustration it is evident that we may find the per cent of gain by use of the formula $g = (S - C) \div C$, in which g = rate of gain, S = selling price, and C = cost. If g turns out to be negative, it indicates a loss.

Exercise 4. Profit and Loss

1. A lawn mower which cost \$4.50 is sold for \$6. What is the per cent of gain, reckoned on the cost?

2. A suit of clothes which cost \$25 is sold for \$24.50. What is the per cent of loss, reckoned on the cost?

3. A merchant buys potatoes at \$1.20 a bushel and sells them at \$1.50 a bushel. What is the per cent of gain, reckoned on the cost?

Cost of doing Business. In computing the cost of goods it is customary to add to the wholesale price, known as the *prime cost*, the freight, drayage, salaries of buyers, and all other expenses incurred in getting the goods ready to sell. These expenses are called *buying expenses.* Such charges as rent, insurance, light, bookkeeping, and interest on the money invested are called *overhead charges* or *overhead.*

In addition to the expenses just enumerated the total expense incurred in selling an article must be considered before the profits of the sale can be computed. To these items must be added the proper proportion of overhead expenses, the total making up the *cost of doing business.*

Illustrative Problem. In a certain business the salaries of buyers, the freight, the drayage, and other direct buying expenses are \$4500 a year. The prorated share of overhead buying expenses, that is, the share to be charged to the general cost of the goods, is \$2500. The prime cost of the purchases for the year is \$84,000. What per cent of the prime cost of each article must be added to the cost in order to cover buying expenses?

$\$4500 + \$2500 = \$7000$, total buying expenses.

$\$7000 \div \$84{,}000 = 8\frac{1}{3}\%$, the per cent to be added.

If the salaries of salesmen and other direct selling expenses are \$6240, the prorated share of overhead selling expenses is \$5360, and the prime cost of goods sold is \$105,000, what per cent of the prime cost must be added to the cost of each article to cover selling expenses?

$\$6240 + \$5360 = \$11{,}600$, total selling expenses.

$\$11{,}600 \div \$105{,}000 = 11.047\%$, the per cent to be added.

Therefore, in order to cover buying and selling expenses the prime cost must be increased by $8\frac{1}{3}\% + 11.047\%$, or 19.38%.

Exercise 5. Cost of Doing Business

1. A dealer buys 40 stoves at \$17.50. He pays \$31.75 freight and sells the stoves at \$26. His selling expenses and overhead are \$1.80 on each stove. What per cent of profit does he make on the total cost?

2. A certain manufacturer adds 20% of the prime cost of an article to cover all buying and selling expenses. The prime cost of each article is \$14.80. At what price must he sell this article in order that he may make a net profit of 20% on the total cost?

3. A printer agreed to supply some advertising cards to a merchant for \$6.80 per hundred. The cost of the material and the direct labor was \$3.40 per hundred, and the overhead was 75¢ per hundred. What per cent of profit on the total cost does the printer make?

4. A manufacturer determines his selling price by adding 28% to the manufacturing cost. What was the cost of manufacturing an article which he sells for \$25.60?

5. A dealer buys some ice chests at \$18.50. His buying and selling expenses and overhead are 18% of the prime cost. He sells the ice chests at \$25. What per cent does he gain on the prime cost? on the total cost?

6. A dry-goods merchant buys 480 yd. of linen at $87\frac{1}{2}$¢. His buying and selling expenses and overhead are 20% of the prime cost. He sells 300 yd. at \$1.20, and the rest at \$1.18. What is his per cent of profit on the total cost?

7. A dealer buys 160 reams of paper at \$1.40. He sells 12 reams, which were damaged, at 90¢, and the rest at \$1.65. His buying and selling expenses and overhead are 12% of the prime cost. What per cent of the total cost does he gain or lose on the transaction?

Profit Reckoned on Selling Price. Business houses often compute the profit on the basis of the selling price. One advantage is that cost is a complex term including prime cost, buying and selling expenses, and overhead, while selling price is a simple term and has but one meaning.

Illustrative Problems. **1.** A grocer makes a profit of 20% on his selling price. Find the profit on sales of \$740.

The profit is 20% of \$740, or \$148.

2. On an article which was sold for \$32 the gain on the selling price was $12\frac{1}{2}\%$. What was the gain on the cost?

$12\frac{1}{2}\%$ of \$32 = $\frac{1}{8}$ of \$32 = \$4, gain.

$\frac{7}{8}$ of \$32 = \$28, cost.

\$4 ÷ \$28 = $14\frac{2}{7}\%$, the per cent of gain on the cost.

Exercise 6. Profit Reckoned on Selling Price

1. A dealer sells at a gain of 20% on the selling price. What is the amount of gain on a sale of \$40.50?

2. If a dealer gains 20% on the selling price, which was 25% above cost, what per cent does he gain on the cost?

3. By selling an article for \$3 a dealer gains $16\frac{2}{3}\%$ on the selling price, but the article is damaged and is sold for \$2.75. What per cent is gained on the selling price?

4. A dealer marked 100 copies of a book \$1.25 each, thereby expecting a net profit of 20% on the selling price. He sold 60 copies at \$1.25 and the rest at 90¢. Find the per cent of gain or loss, reckoned on the cost.

5. A dealer buys some desks at a prime cost of \$22 each. The buying and selling expenses and overhead charges are 12% of the prime cost. At what price must each desk be sold so as to gain 25% on the selling price?

Commercial Discount. We shall now briefly review commercial discount, and shall first consider the following example of a bill with a single discount:

Kansas City, Mo., April 19, 1922

Arnold and Fisher
Omaha, Nebraska

Bought of THE GENERAL SUPPLY CO.

April	19	7 doz. door handles, No. 487	3.00	21	
		5 doz. hooks, No. 126 A	1.60	8	
				29	
			40%	11	60
				17	40

It is evident that the net price is the list price less the discount. Using initial letters as usual, we have $n = l - d$.

Exercise 7. Commercial Discount

1. Since discount is a given per cent of the list price, the above formula may be stated in terms of the list price (l) and the rate of discount ($r\%$). Write the formula.

2. Find the list price and net price of the following bill of goods dated Sept. 16 and paid Sept. 25, terms 2/10, *N*/30: 9 cases peas @ $1.60, 7 cases soap @ $5.10.

The symbols 2/10, *N*/30 mean 2% discount if paid in 10 da.; net (meaning list price in such cases) after that, payment due in 30 da.

3. Find the net price of a chair marked $20.50 but offered at 20/10, *N*/30.

Several Discounts. In Book I you learned that two or more discounts are allowed in some kinds of business. For example, a furniture dealer may buy desks from a wholesale dealer with discounts of 20, 30, that is, of 20% and 30%. This means that one of these discounts is first deducted and then the other is deducted from the remainder. It makes no difference in the result whether 20% or 30% is deducted first.

For example, if a furniture dealer buys 6 oak desks at $42.50, less 20, 30, what is the net price? The solution may be given as follows:

$42.50	5)$255	$255	$204	$204
6	$51	51	.30	61.20
$255.00		$204	$61.20	$142.80

It is evident, however, that we may shorten this solution. For example, we found 20% of $255 by dividing by 5, and then subtracted the result from $255, whereas it would have been shorter simply to take 80% of $255, thus finding $204 at once. Similarly, we took 30% of $204 and subtracted the result from $204, but it would have been shorter to take 70% of $204 at once.

Furthermore, since we wish to find 70% of 80% of $255, we really wish to find 56% of $255, and this will leave 44% of $255, which is $112.20, the total discount. This 44% is more easily found by the following simple rule:

To find a single rate equivalent to two successive discount rates, from the sum of the two rates subtract their product.

In the above case, from 0.20 + 0.30, or 0.50, subtract 0.20 × 0.30, or 0.06. The result is 0.44, the single rate of discount.

Algebraically expressed, $(1 - r)(1 - r') = 1 - (r + r' - rr')$.

Exercise 8. Several Discounts

1. Express as a formula the statement about several discount rates as given on page 117.

2. What difference is there between a discount of 25% and 16% and a discount of 16% and 25%? Illustrate by an example.

3. If a dealer buys goods listed at \$80.50 less 20, 5 (that is, 20%, 5%), what is the net price?

4. A wholesale dealer allows discounts of 25, 15. A clerk sold an invoice of \$95 and gave a single discount of 40%. How much did this error cost him or his employer?

5. A jobber lists a plow at \$45 less 20%. A competitor lists the same style of plow for \$48 less $33\frac{1}{3}$%. In order to meet his competitor's price exactly, what additional discount must the first jobber give?

Make out bills for the following:

6. 460 yd. taffeta @ \$1.60; 540 yd. velvet @ \$1.80. Discounts 25%, 10%.

7. 9 doz. pairs hinges @ \$4.85; 48 doz. table knives @ \$8.80. Discounts 16%, 12%.

8. 36 doz. locks @ \$4.80; 10 doz. mortise locks @ \$4.80. Discounts 20%, 12%.

9. 580 yd. taffeta @ \$1.08; 9 gross pompons @ \$156; 8 doz. pieces braid @ \$24.60. Discounts 12%, 3%.

10. 720 yd. silk @ \$1.80; 940 yd. lawn @ 28¢; 560 yd. taffeta @ \$1.06. Discounts 12%, 4%, 2%.

11. 250 yd. carpet @ \$1.15; 40 rugs @ \$7.75; 876 yd. carpet @ \$1.37; 320 yd. hangings @ \$1.40; 240 yd. oilcloth @ \$0.48. Discounts 8%, 6%, 6%.

Short Methods. As you learned in Book I, and also on pages 88 and 89 of this book, there are numerous short practical methods of working with numbers. He would be a poor bookkeeper, for example, who always solved an example like that of finding the cost of 3280 yd. of cloth at $37\frac{1}{2}$¢ a yard by long multiplication. He should see at once that the answer is simply $\frac{3}{8}$ of \$3280, or $3 \times \$410$, or \$1230. Some of the important short methods not considered on pages 88 and 89 will now be reviewed.

1. *To multiply an integer by a power of 10, annex as many zeros to the multiplicand as there are zeros in the multiplier.*

For example, $10 \times 37 = 370$, and $100 \times 250 = 25{,}000$. As already learned, 10 is the first power of 10; 100 is the second power; and so on.

2. *To multiply a decimal by a power of 10, move the decimal point to the right as many places as there are zeros in the multiplier, annexing zeros if necessary.*

For example, $100 \times 0.134 = 13.4$, and $1000 \times 31.72 = 31{,}720$.

3. *To divide a number by a power of 10, move the decimal point to the left as many places as there are zeros in the divisor, prefixing zeros if necessary.*

For example, $372 \div 100 = 3.72$, and $4.27 \div 1000 = 0.00427$.

Every integer may be thought of as having a decimal point written after it; that is, $372 = 372. = 372.0$, and so on.

4. *To multiply by 30, first multiply by 3 and then move the decimal point one place to the right; and similarly for any other multiple of a power of 10.*

For example, $30 \times 178 = 10 \times (3 \times 178) = 10 \times 534 = 5340$.

5. *To divide by 30, first divide by 3 and then move the decimal point one place to the left; and similarly for any other multiple of a power of 10.*

For example, $1530 \div 30 = (1530 \div 3) \div 10 = 510 \div 10 = 51$.

Aliquot Parts. One of the most important of the short methods of multiplying or dividing depends upon a knowledge of aliquot parts. You already know some of the aliquot parts of 100, as that 50 is $\frac{1}{2}$ of 100, $33\frac{1}{3}$ is $\frac{1}{3}$ of 100, and so on; but in trade we need to know others, even $\frac{1}{7}$, $\frac{1}{9}$, and $\frac{1}{11}$ being occasionally used.

The following table should be learned:

$0.50 = \frac{1}{2}$	$0.14\frac{2}{7} = \frac{1}{7}$	$0.08\frac{1}{3} = \frac{1}{12}$
$.33\frac{1}{3} = \frac{1}{3}$	$.12\frac{1}{2} = \frac{1}{8}$	$.06\frac{2}{3} = \frac{1}{15}$
$.25 = \frac{1}{4}$	$.11\frac{1}{9} = \frac{1}{9}$	$.06\frac{1}{4} = \frac{1}{16}$
$.20 = \frac{1}{5}$	$.10 = \frac{1}{10}$	$.05 = \frac{1}{20}$
$.16\frac{2}{3} = \frac{1}{6}$	$.09\frac{1}{11} = \frac{1}{11}$	$.04 = \frac{1}{25}$

As already explained in Book I, zero may be written before a decimal point in a case like 0.50, or it may be omitted. Computers usually find it safer to write it, so that the decimal point will not be overlooked; that is, \$0.25 is less likely to be mistaken than \$.25. When the figures are in columns, however, as in the above case, it is sufficient to write the zero only at the top, since the eye is then almost sure to notice the decimal points below.

The following table should also be learned:

$0.66\frac{2}{3} = \frac{2}{3}$	$0.40 = \frac{2}{5}$	$0.37\frac{1}{2} = \frac{3}{8}$
$.75 = \frac{3}{4}$	$.60 = \frac{3}{5}$	$.62\frac{1}{2} = \frac{5}{8}$
$.83\frac{1}{3} = \frac{5}{6}$	$.80 = \frac{4}{5}$	$.87\frac{1}{2} = \frac{7}{8}$

For example, study the following:

1. $33\frac{1}{3} \times 96$	$33\frac{1}{3} = \frac{1}{3}$ of 100	$\frac{1}{3}$ of $9600 = 3200$
2. $6\frac{1}{4} \times 480$	$6\frac{1}{4} = \frac{1}{16}$ of 100	$\frac{1}{16}$ of $48{,}000 = 3000$
3. $14 \div 12\frac{1}{2}$	$12\frac{1}{2} = \frac{1}{8}$ of 100	$8 \times 0.14 = 1.12$
4. $320 \div 0.62\frac{1}{2}$	$0.62\frac{1}{2} = \frac{5}{8}$	$\frac{8}{5}$ of $320 = 512$

All these cases being reviews of cases in Book I and in earlier work in arithmetic, we shall simply give enough practice work to make sure that the process is familiar to the students.

Illustrative Problems. 1. How much will it cost to manufacture 4800 yd. of cloth at $87\frac{1}{2}$¢ a yard?

You should simply think that if the cloth cost $1 a yard, the cost would be $4800, so that the cost is really $\frac{7}{8}$ of $4800, or $4200.

2. A manufacturer sells a lot of chairs for $800, the net price being $6.25 each. How many did he sell?

You should remember that $0.62\frac{1}{2} = \frac{5}{8}$, and so $\$6.25 = \frac{5}{8}$ of $10. Then to divide $800 by $6.25 is the same as to divide it by $\frac{5}{8}$ of $10, or to divide 80 by $\frac{5}{8}$, or to multiply 80 by $\frac{8}{5}$, so that the answer is 128.

Exercise 9. Short Methods

Examples 1 to 18, oral

Multiply, in turn, by 10, by 100, and by 1000:

1. 45. **3.** 7.5. **5.** 0.75. **7.** $2\frac{1}{2}$. **9.** $3\frac{3}{4}$.

2. 68. **4.** 6. **6.** 3.85. **8.** $7\frac{1}{4}$. **10.** $5\frac{4}{5}$.

Divide, in turn, by 10, by 100, and by 1000:

11. 87. **13.** 360. **15.** 82.5. **17.** 3972.

12. 75. **14.** 475. **16.** 7.65. **18.** 487.5.

Multiply, in turn, by 25, by $2\frac{1}{2}$, and by 0.25:

19. 76. **20.** 83.6. **21.** 1.44. **22.** 9.6. **23.** 0.8.

Divide, in turn, by $0.12\frac{1}{2}$, by $12\frac{1}{2}$, and by 125:

24. 75. **25.** $86\frac{1}{2}$. **26.** 92.8. **27.** 725. **28.** 0.5.

Multiply or divide as indicated:

29. $16\frac{2}{3} \times 720$. **32.** $64 \times 62\frac{1}{2}$¢. **35.** $\$75 \div 16\frac{2}{3}$.

30. $14\frac{2}{7} \times 490$. **33.** $72 \times 37\frac{1}{2}$¢. **36.** $\$85 \div 83\frac{1}{3}$.

31. $83\frac{1}{3} \times 360$. **34.** $48 \times 87\frac{1}{2}$¢. **37.** $\$25 \div 0.12\frac{1}{2}$.

Further Short Methods. The following short methods will also be found very useful:

1. *To multiply by 9, first multiply by 10 and from the result subtract the multiplicand.*

For we see that, if m represents the multiplicand, we have $10\,m - m = 9\,m$. That is, if from 10 times the number we take once the number, we have 9 times the number.

Thus the multiplication of 754 by 9 is here shown.

$$\begin{array}{r} 7540 \\ \underline{754} \\ 6786 \end{array}$$

2. *To multiply by 99, first multiply by 100 and from the result subtract the multiplicand.*

For we see that, if m represents the multiplicand, we have $100\,m - m = 99\,m$. Thus the multiplication of 473 by 99 is here shown.

$$\begin{array}{r} 47,300 \\ \underline{473} \\ 46,827 \end{array}$$

Similarly, in the multiplication of a number by 999, we have $1000\,m - m = 999\,m$, from which the rule is easily derived.

Exercise 10. Short Methods

Examples 1 to 12, oral

Multiply, in turn, by 9 and by 99:

1. 15.	**3.** 32.	**5.** 57.	**7.** 75.	**9.** 88.
2. 25.	**4.** 41.	**6.** 63.	**8.** 78.	**10.** 96.

11. State a rule for multiplying by 999.

12. State a rule and formula for multiplying by 9999.

Multiply, in turn, by 9 and by 99:

13. 7.8.	**15.** 927.	**17.** 7436.	**19.** 576.23.
14. 6.3.	**16.** 689.	**18.** 2987.	**20.** 498.48.

21. Multiply \$7436.75 by 999; \$6572.36 by 999.

22. How much is $5723.25 \times \$999$? 27.37×9999?

Numbers ending in 5. There are two convenient methods for finding the product of two numbers ending in 5.

If the sum of the parts at the left of the 5's is even, to their product add half their sum and annex 25.

For example, multiply 65 by 45.

$4 \times 6 =$	24,	product of digits at the left
$\frac{1}{2}$ of $(4+6) =$	5,	half their sum
	2925,	sum with 25 annexed

Knowing algebra as you do, there is nothing mysterious about this. Indeed, the rule is found by algebra, as you will see if you multiply $6t + 5$ by $4t + 5$.

If the sum of the parts at the left of the 5's is odd, to their product add half their sum, dropping the $\frac{1}{2}$, and annex 75.

For example, multiply 95 by 45.

$4 \times 9 =$	36,	product of digits at the left
$\frac{1}{2}$ of $(4+9) =$	6,	half their sum, dropping the $\frac{1}{2}$
	4275,	sum with 75 annexed

Exercise 11. Numbers ending in 5

Examples 1 to 16, oral

State the products of the following:

1. 15×35.
2. 15×55.
3. 95×35.
4. 35×45.
5. 35×55.
6. 45×85.
7. 75×95.
8. 85×85.
9. 15×25.
10. 15×65.
11. 35×45.
12. 35×65.
13. 45×75.
14. 55×85.
15. 65×75.
16. 75×85.
17. 45×125.
18. 35×125.
19. 65×225.
20. 75×305.
21. 125×125.
22. 125×135.
23. 235×235.
24. 365×445.

Multiplication by Factors. **1.** Multiply 78 by 45.

Instead of multiplying by 5, then multiplying by 4 tens, and then adding the products, we may notice that $45 = 5 \times 9$. Hence we may multiply by 9, the product being $780 - 78$, or 702; and then we may multiply 702 by 5, or divide 7020 by 2, the result being 3510. We thus save the addition of partial products.

$$\begin{array}{r} 702 \\ 3510 \end{array}$$

2. Multiply 527 by 486.

Instead of making three separate multiplications and then adding the results, we may notice that $48 = 8 \times 6$, so that the multiplier is really 6 and 8×6. We then proceed as here shown:

$$\begin{array}{rl} 527 & \\ 486 & \\ \hline 3\,162 & = 6 \times 527 \\ 252\,96 & = 8 \times 3162, \text{ or } 48 \times 527 \\ \hline 256{,}122 & \end{array}$$

This saves us one multiplication and some work in addition.

3. Multiply 624 by 237.

The result is the same if we multiply 237 by 624. This is a convenient change, because we see that in 624 we have $24 = 4 \times 6$. We then proceed as here shown:

$$\begin{array}{rl} 237 & \\ 624 & \\ \hline 142\,200 & = 6 \times 237, \text{ written as 100's} \\ 5\,688 & = 4 \times 1422, \text{ or } 24 \times 237 \\ \hline 147{,}888 & \end{array}$$

Again we save one multiplication and some work in addition.

Exercise 12. Short Methods

Examples 1 to 19, oral

State the results of the following:

1. $32 \times 0.06\frac{1}{4}$.
2. $75 \times 0.33\frac{1}{3}$.
3. $40 \times 0.62\frac{1}{2}$.
4. $80 \times 0.87\frac{1}{2}$.
5. $24 \div 0.5$.
6. $25 \div 0.33\frac{1}{3}$.
7. $20 \div 0.66\frac{2}{3}$.
8. $30 \div 0.37\frac{1}{2}$.
9. $750 \div 100$.
10. $2575 \div 1000$.
11. $346.4 \div 100$.
12. $0.75 \div 10$.

State the cost of each of the following items:

13. $12\frac{1}{2}$ lb. cocoa at 64¢.

14. $6\frac{1}{4}$ gross buttons at 48¢.

15. 25 yd. cloth at 28¢; at 36¢; at 48¢.

16. $6\frac{1}{4}$ bu. wheat at $1.12.

To divide 112 by 16, first divide by 2 and then by 8.

17. State a short method of multiplying any number by $12\frac{1}{2}$; by $6\frac{1}{4}$; by $8\frac{1}{3}$; by $37\frac{1}{2}$; by 375.

18. State a short method of dividing any number by $33\frac{1}{3}$; by $3\frac{1}{3}$; by $66\frac{2}{3}$; by 25; by $2\frac{1}{2}$.

19. What aliquot part of 100 is $12\frac{1}{2}$? 25? $6\frac{1}{4}$? $33\frac{1}{3}$?

As on page 89, find the products of the following:

20. 99×101.
21. 97×103.
22. 199×201.
23. 198×202.
24. 1999×2001.
25. 1998×2002.

As on page 88, find the values of the following:

26. 17^2. **27.** 23^2. **28.** 34^2. **29.** 46^2. **30.** 71^2.

Perform the following multiplications as on page 124:

31. 255×3786.
32. 328×4179.
33. 497×62.83.
34. 981×7.076.
35. 742×58.72.
36. 7.63×9.826.

Foreign Money. Every country has its own system of money, although several European countries use systems which are very much the same as that of France. On account of our trade with foreign countries there are a few names of coins which everyone should know. The following table shows the most important units of value:

The British pound (£) is equal to \$5 (\$4.8665).
The British shilling (s.) is equal to 25¢ (24.33¢).
The German mark (M.) is equal to 25¢ (23.8¢).
The French franc (fr.) is equal to 20¢ (19.3¢).
The Italian lira (£) is equal to 20¢ (19.3¢).
The Russian ruble (R.) is equal to 50¢ (51.5¢).

British money is also called sterling money. One pound is written either £1 or 1 l., and £1 = 20 s. The gold coin, equal in value to a pound, is called a sovereign. The symbol £ is used in Great Britain for pound (value) and in Italy for lira.

The word *lira* is pronounced lee′rah. The plural is *lire* (lee′ray).

Our relations with Japan and Mexico make it convenient to know that 1 yen (Japanese) = 1 peso (Mexican) = 50¢ (49.8¢).

The equivalents given in parentheses should be used in the written exercises but not memorized. The others are sufficient for ordinary mental computation and may be used in the oral problems.

Exercise 13. Foreign Money

Examples 1 to 38, oral

1. A Liverpool boy writes that he has just spent £3 for a suit of clothes. This is how many dollars?

2. A Chicago boy writes to a friend in London that he has just spent \$5.25 for a baseball suit. The London boy thinks of this as how many pounds and shillings?

3. Fred reads that a good tennis racket costs 9 s. in England. He thinks of this as how many dollars and cents?

4. There are 12 pence (pennies) in a shilling. A penny is equal to about how many of our cents?

5. If you were in England and paid 7 d. (7 pence) for a pencil, about how many cents would you pay?

6. Karl's cousin from Berlin shows him a 10-mark gold piece. Karl thinks of this as how many dollars and cents?

7. Karl's cousin shows him a silver piece marked 2 M. Karl thinks of this as how many cents?

8. Charles has a cousin from Paris visiting him. The cousin shows him a 20-franc gold piece and a silver piece marked 1 fr. How much is each of these coins worth in American money?

9. When his uncle landed from Genoa, Tony met him. His uncle asked Tony about how much he should receive for 50 lire. What should Tony tell him?

10. When Ivan's aunt came over from Russia she had 300 R. in Russian money which she wished to have changed into American money. About how much should she receive in exchange for her 300 R.?

State the approximate equivalents of the following:

11. £ 9.	**18.** £ 9.	**25.** 40 M.	**32.** 30 fr.
12. £ 20.	**19.** £ 20.	**26.** 80 M.	**33.** 50 fr.
13. £ 35.	**20.** £ 40.	**27.** 75 M.	**34.** 200 fr.
14. 15 s.	**21.** £ 55.	**28.** 200 M.	**35.** 600 fr.
15. £ 6 10 s.	**22.** £ 75.	**29.** 500 M.	**36.** 4000 fr.
16. £ 12 2 s.	**23.** £ 80.	**30.** 700 M.	**37.** 200 R.
17. £ 35 5 s.	**24.** £ 60.	**31.** 740 M.	**38.** 2000 R.

Exs. 11–17 refer to English money; Exs. 18–24 to Italian money.

39. A druggist imported a case of drugs from Germany. The bill was 3650 M. Express this amount in our money.

In all this written work use the exact equivalents given on page 126, without regard to current rates of exchange.

40. A dealer bought 2400 yd. of carpet in England at 5 s. a yard. Find the total cost in American money.

41. A dealer in London cabled an offer for 600 automobiles at £225. The manufacturer had to think of this in dollars and cents before he knew whether to accept. Find in our money the offer per car and the total offer.

42. A city milliner in our country imported a lot of hats from Paris. The invoice amounted to 7850 fr. Express this amount in our money.

43. A man traveling in Italy bought a piece of statuary for 3250 lire and a vase for 235 lire. Find the total cost in American money.

44. A Russian firm placed an order in the United States for 35 machines at 575 R. each. Compute the total amount of this order in our money.

45. A traveler bought a picture in Berlin for 8750 M. How much is the purchase price in our money?

Express the following in United States money:

46. £90.	**52.** £350.	**58.** 480 M.	**64.** 8500 fr.
47. £46.	**53.** £425.	**59.** 586 M.	**65.** 8325 fr.
48. 19 s.	**54.** £3250.	**60.** 792 M.	**66.** 6985 fr.
49. £9 14 s.	**55.** £4750.	**61.** 8345 M.	**67.** 8282 fr.
50. £8 17 s.	**56.** £5275.	**62.** 9175 M.	**68.** 6025 R.
51. £6 12 s.	**57.** £6760.	**63.** 9500 M.	**69.** 3285 R.

Exs. 46–51 refer to English money; Exs. 52–57 to Italian money.

Metric System. In Book I we studied the metric system, this system being used in a large part of the world, and we need to know how to use it in our foreign trade. We also need it for our work in such subjects as electricity and in many kinds of manufacturing. We shall now review the tables that are of most importance in the arithmetic of trade, namely, the tables of length, weight, and capacity.

Table of Length. The table of length is as follows:

A myriameter	=	10,000 meters	
A **kilometer** (km.)	=	1000 meters	
A hektometer	=	100 meters	
A dekameter	=	10 meters	
Meter (m.), about 39.37 in.			
A decimeter (dm.)	=	0.1	of a meter
A **centimeter** (cm.)	=	0.01	of a meter
A **millimeter** (mm.)	=	0.001	of a meter

10 millimeters = 1 centimeter ; 10 centimeters = 1 decimeter

The decimeter is equal to about 4 in., and the kilometer is equal to about 0.6 mi., or $\frac{3}{5}$ mi.

The abbreviations in this book are recommended by various scientific associations. Some writers, however, use Km., cm., and mm., for kilometer, centimeter, and millimeter, the principle being to use capital letters in the abbreviations of multiples of the principal unit.

Only the measures printed in heavy type in these tables are commonly used.

One advantage of the metric system is that to change from one measure of length, weight, or capacity to another we simply move the decimal point just as we do in reducing dollars to cents, or cents to dollars. That is, 2 m. = 200 cm. = 2000 mm., and 3 km. = 3000 m. = 3,000,000 mm.

Table of Weight. The table of weight is as follows:

A **metric ton** (t.)	=	1,000,000 grams
A quintal (q.)	=	100,000 grams
A myriagram	=	10,000 grams
A **kilogram** (kg.)	=	1000 grams
A hektogram	=	100 grams
A dekagram	=	10 grams
Gram (g.), about 15.43 grains		
A decigram	=	0.1 of a gram
A centigram (cg.)	=	0.01 of a gram
A milligram (mg.)	=	0.001 of a gram

A kilogram, commonly called a *kilo*, is about 2.2 lb. A 5-cent piece weighs 5 g. A metric ton is nearly 2204.6 lb., and so it is about 10% more than our common ton.

Table of Capacity. The table of capacity is as follows:

A **hektoliter** (hl.)	=	100 liters
A dekaliter	=	10 liters
Liter (l.), about 1 qt.		
A deciliter	=	0.1 of a liter
A centiliter (cl.)	=	0.01 of a liter
A milliliter (ml.)	=	0.001 of a liter

A liter is the same as a cubic decimeter. A liter is so nearly equal to a quart that we commonly think of the liter and quart as the same. In reality a liter is equal to 0.91 of a dry quart or 1.05 liquid quarts, but these facts need not be memorized.

Only the liter and hektoliter are in common use. The centiliter is used more often than the deciliter and milliliter.

It is much easier to remember these relations than to remember the relations of gills, pints, quarts, and gallons.

Prefixes. From the tables we have just studied we see that the following are the meanings of the prefixes in the tables of length, weight, and capacity:

PREFIX	MEANS	AS IN	WHICH	MEANS
myria-	10,000	myriameter	10,000	meters
kilo-	1000	kilogram	1000	grams
hekto-	100	hektoliter	100	liters
deka-	10	dekameter	10	meters
deci-	0.1	decimeter	0.1	of a meter
centi-	0.01	centigram	0.01	of a gram
milli-	0.001	millimeter	0.001	of a meter

Exercise 14. Metric System

Express the following as meters:

1. 60 km. **2.** 3800 cm. **3.** 52,500 mm. **4.** 9.8 km.

Express the following as centimeters:

5. 3 m. **6.** 50 mm. **7.** 2 km. **8.** 36.6 m.

Express the following as millimeters:

9. 3 m. **10.** 6.4 cm. **11.** 9.6 m. **12.** 2 km.

Express the following as kilometers:

13. 9000 m. **14.** 8750 m. **15.** 25,000 cm. **16.** 3.4 m.

Express the following as grams:

17. 4.3 kg. **18.** 0.8 kg. **19.** 8000 mg. **20.** 60 cg.

Express the following as liters:

21. 4 hl. **22.** 4.5 hl. **23.** 500 cl. **24.** 0.45 hl.

25. A merchant wishes 1000 yd. of French silk. How many meters of it must he order?

26. The distance between two forts is 20 km. What is the distance in miles?

27. A siege gun has a diameter of 40 cm. What is the diameter in inches?

28. If a horse weighs 720 kg., how many pounds does he weigh?

29. The distance by rail from Boston to Chicago by one route is 1033 mi. How many kilometers is this?

30. The Eiffel Tower in Paris is about 300 m. high. Express this height in feet.

31. Sound travels 332 m. per second. Two forts are 5.5 km. apart. How long after a big gun is fired in one of the forts before the report is heard at the other?

32. The average weight of girls $13\frac{1}{2}$ yr. old is 40.23 kg.; of boys, 38.46 kg. Express these weights in pounds.

33. The average lung capacity of a girl 13 yr. of age is 1827 cm^3.; of a boy at the same age, 2108 cm^3. Taking 1 cm^3. (1 cubic centimeter) as 0.061 cu. in., express each average lung capacity in cubic inches.

34. George Williams, while visiting in Paris, walked 14.5 km. one day. How many miles did he walk?

35. The average right-hand grip of a girl 13 yr. of age is 21.84 kg.; of a boy, 24.44 kg. Express each in pounds.

36. A German farmer has a farm of 62.35 hektares. A hektare is 2.47 A. Express the area of the farm in acres to the nearest hundredth.

37. A train which travels 1.2 km. per minute travels what fraction of a mile per minute?

Exercise 15. Cumulative Review

1. M. R. Flemming of Kansas City bought of C. R. Briggs & Co., Chicago, the following invoice: 12 Wilton rugs @ \$48; 8 Turkish rugs @ \$17.50; 240 yd. Brussels carpet @ \$1.45. Discounts of $12\frac{1}{2}\%$ and 10% were allowed on the bill. Write, date, and receipt the bill, and compute the net amount due.

2. A dealer in Louisville imported from England goods valued at £420 8 s. Transportation and all other expenses incurred in buying and selling the goods amounted to 40% of the original value. How much must he receive for the goods in order to gain 20% on the total cost?

Allow \$4.87 to the pound in Exs. 2 and 3.

3. A wholesale merchant in Pittsburgh imported 2000 doz. pairs of cotton stockings valued at £445 8 s. The entire cost of buying and selling the goods was 30% of the invoice. At what price per pair must the stockings be sold to gain $33\frac{1}{3}\%$ on the total cost?

4. If you lay off a square meter upon the floor and within this area lay off a square yard, the first area is how many times the second?

5. A druggist buys 1 kg. of quinine for \$11 and puts it into capsules of 0.1 g. each. Find to the nearest tenth of a cent the cost of the quinine in each capsule.

6. An express train in Germany runs at the rate of 80 km. per hour. A train in the United States runs at the rate of 50 mi. per hour. The speed of the faster train is what per cent greater than that of the slower train?

Teachers will find at the end of each large topic a cumulative review, that is, a review covering certain topics in the preceding work in arithmetic, and also a series of problems without numbers.

Exercise 16. Problems without Numbers

1. If you are told the number of feet of molding purchased, how do you find the number of yards?

2. If you are told the weight of a box in kilograms, how do you find its weight in pounds?

3. How do you find a single discount equivalent to two successive discounts? to three successive discounts?

4. If you are told the cost of some goods in pounds sterling, how do you find the cost in United States money?

5. Can the discount equivalent to two given discounts ever be equal to their sum? State the reason.

6. If you buy goods at a given discount on the list price and sell them at an advance of the same per cent on the net cost, are they sold for more than or less than the list price? State the reason.

7. If you know the cost of some goods in marks, how do you express the cost in United States money? How could you express the cost in francs?

8. If you have no scales arranged on the metric system, what must you know and what must you do to determine the weight of some goods in kilograms?

9. State how you might proceed to find the extreme length and extreme width of your county in kilometers.

10. If you are told the dimensions of a rectangular bin in meters, how can you find the number of hektoliters of wheat that it would hold when just full?

11. If you wish to send to a friend in France a certain number of francs by money order, and know the fee charged, how do you find the entire amount that you must pay for the order?

II. ARITHMETIC OF TRANSPORTATION

Passenger Rates. In general, for railway tickets between stations the rate is a certain number of cents per mile, usually 2¢, $2\frac{1}{2}$¢, or 3¢, this rate being fixed by law.

The details of passenger rates vary considerably and are of no particular interest to the school. It is desirable, however, that teachers or students should ascertain the rate in their state and find the cost of tickets to certain places, this cost being ascertained from the mileage given in railway folders, as in Exs. 1 and 2 below.

The school need not consider such exceptional cases as those of competing lines, excursion tickets, monthly tickets to suburban points, and reduced-rate tickets. The large question, indicated by the types in Exs. 1 and 2, is the chief concern.

Exercise 17. Passenger Rates

1. The passenger rate in Minnesota is 2¢ a mile. What is the cost of a railway ticket from St. Paul to Duluth if the distance is 153 mi.?

2. The rate in Illinois being 2¢ a mile, find the cost of tickets from Chicago to each of the following places: Mattoon, 171 mi.; Carbondale, 307 mi.; Cairo, 364 mi.

3. In a state where the rate is 3¢ a mile a man buys a ticket to a place 127 mi. away in the state, giving the agent a $5 bill. How much change is due with the ticket?

4. In a state where the rate is 2¢ a mile a man buys a ticket to a place 378 mi. away in the state and pays $2 for a ticket for a sleeping-car berth. Find the total cost.

5. The rate between two cities is $5.25, but on a limited train it is $1 more, besides which there is a charge of $1.25 for a parlor-car seat. A man buys tickets for himself and wife on the limited train and hands the agent two $10 bills. How much change is due with the tickets?

Freight Rates. Bulky goods are usually sent by freight, the rate being stated in cents per 100 lb. or sometimes in dollars and cents per ton. The rate depends on the expense of handling and on the risk of loss or damage.

The following is a portion of a tariff, showing the number of cents per 100 lb. for freight of six different classes shipped from Indianapolis to various points:

Class	1	2	3	4	5	6
Rock Island, Ill.	42.9	40.6	30.7	21.3	18.7	14.5
Columbus, Ohio	31.5	27.3	22.1	14.2	11.6	9.5
Cleveland, Ohio	42.0	35.7	26.3	17.9	15.2	12.1
Dayton, Ohio	25.7	23.1	20.5	13.1	9.5	8.4
Toledo, Ohio	38.9	33.6	24.7	16.8	13.7	10.5

It is desirable to have the students ascertain the rates from their city to various places. The railways will usually furnish a school with a pamphlet giving the classification of freight and the local rates. These rates are usually subject to approval by state authorities. Small details need not be considered, but the meaning of the expression f.o.b. (free on board, that is, delivered on board the freight car) and c.l. (carload lots, where the rate is lower) should be stated.

Exercise 18. Freight Rates

Using the above table, find the charges on the following:

1. 14,260 lb. third class and 2420 lb. first class, from Indianapolis to Columbus; to Rock Island; to Dayton.

2. 175 lb. second class and 350 lb. fourth class, from Indianapolis to Cleveland; to Dayton; to Toledo.

3. 26,320 lb. furniture, which is in the first class, from Indianapolis to Columbus.

4. 48,250 lb. fifth class, from Indianapolis to Dayton.

Express Rates. The express classification is simpler than the freight, most articles being rated as first class. Agents have books giving the scale number to be used in computing the rates to every other express station, these scale numbers running from 1 to 294, according to the distance. These books also give the rates for each scale number and for various weights from 1 lb. to 100 lb. The following is a portion of such a schedule, showing the rates for a few scale numbers only and for shipments weighing from 25 lb. to 30 lb.:

Pounds	25		26		27		28		29		30		31		32		Pounds
	Class 1	Class 2	Class 1	Class 2	Class 1	Class 2	Class 1	Class 2	Class 1	Class 2	Class 1	Class 2	Class 1	Class 2	Class 1	Class 2	
25	59	45	60	45	61	46	62	47	64	48	65	49	66	50	67	51	25
26	60	45	62	47	63	48	64	48	65	49	67	51	68	51	69	52	26
27	62	47	63	48	65	49	66	50	67	51	69	52	70	53	71	54	27
28	63	48	65	49	66	50	68	51	69	52	70	53	72	54	73	55	28
29	65	49	66	50	68	51	69	52	71	54	72	54	74	56	75	57	29
30	66	50	68	51	69	52	71	54	72	54	74	56	75	57	77	58	30

Exercise 19. Express Rates

All work oral

State the cost of sending by express parcels of the following weights to places having the scale numbers as given:

1. 28 lb., scale number 28, first class, and 30 lb., scale number 32, second class.

2. 25 lb., scale number 26; 27 lb., scale number 31; 28 lb., scale number 28; all first class.

3. 26 lb., scale number 29; 28 lb., scale number 25; 30 lb., scale number 27; all second class.

Parcel Post. Certain articles and products may be sent by parcel post. The combined length and girth of a parcel cannot exceed 84 in., and a fraction of a pound is counted a full pound. The following is the present table of rates for parcels up to 10 lb. sent by parcel post in this country:

Weight in Pounds	Local	Zones							
		1st Up to 50 miles	2d 50 to 150 miles	3d 150 to 300 miles	4th 300 to 600 miles	5th 600 to 1000 miles	6th 1000 to 1400 miles	7th 1400 to 1800 miles	8th Over 1800 miles
1	$0.05	$0.05	$0.05	$0.06	$0.07	$0.08	$0.09	$0.11	$0.12
2	.06	.06	.06	.08	.11	.14	.17	.21	.24
3	.06	.07	.07	.10	.15	.20	.25	.31	.36
4	.07	.08	.08	.12	.19	.26	.33	.41	.48
5	.07	.09	.09	.14	.23	.32	.41	.51	.60
6	.08	.10	.10	.16	.27	.38	.49	.61	.72
7	.08	.11	.11	.18	.31	.44	.57	.71	.84
8	.09	.12	.12	.20	.35	.50	.65	.81	.96
9	.09	.13	.13	.22	.39	.56	.73	.91	1.08
10	.10	.14	.14	.24	.43	.62	.81	1.01	1.20

Further details may be obtained by inquiry at any post office.

Exercise 20. Parcel Post

Compute the charge on each of the following by parcel post:

1. 6 lb. 3 oz., local.
2. 8 lb. 4 oz., local.
3. 9 lb. 1 oz., local.
4. 7 lb. 10 oz., local.
5. 8 lb. 7 oz., to 4th zone.
6. 5 lb. 9 oz., to 8th zone.
7. 7 lb. 2 oz., to 3d zone.
8. 9 lb. 10 oz., to 5th zone.

9. Each week a farmer sends a package of butter weighing 3 lb. 7 oz. to his son, who lives in a city 75 mi. distant. Find the cost of sending the butter by parcel post for 16 wk.

Exercise 21. Cumulative Review

1. Illustrate the use of short methods in the following cases: 125×7840; $87\frac{1}{2}\%$ of 960; $721 \div 33\frac{1}{3}$; 50% of \$1200; 75% of \$1600; $222 \div 66\frac{2}{3}\%$.

2. If a locomotive weighing 115.5 T. can exert a pull of $22\frac{1}{4}\%$ of its weight in starting a train, how many pounds of pull can it exert?

3. What is the discount on a bill of goods for \$775 at 20%, 8%? What is the net amount?

4. A 20,000-ton ship carries 12 times as much freight as an old-style steamer, makes 25% better time, and its running expenses are only 4 times as much. In 1 yr. such a modern ship can do how many times the work of the old type? At the same cost it will carry how many times as much freight?

5. Express £14 9 s. as dollars and cents.

6. A large freight steamer can carry 80 barge loads of grain at 8000 bu. to a barge, while a freight car can carry 1280 bu. How many such freight cars would be needed to carry enough grain to load this steamer?

7. Express 245 km. as miles.

8. A certain steamer burns 960 T. of coal a day, on the average. How many tons of coal must be put in her bunkers for a voyage of $5\frac{1}{2}$ da., allowing a surplus of 2200 T. for emergencies?

9. The rate on a certain class of freight from Boston to Chicago is 36¢ per 100 lb. by rail, or \$5.54 per ton by rail and lake. If a shipper wishes to send 4500 lb., how much does he save by shipping it by rail and lake instead of by rail? What per cent does he save?

Exercise 22. Problems without Numbers

1. Knowing the passenger rate per mile for railway travel in your state and the distance to a certain station, how do you determine, without asking the agent, how much a ticket to that station will cost?

2. If you were a clerk in the post office, how would you proceed to find the postage due on a box of merchandise to be sent by parcel post?

3. If you were an express agent, how would you proceed to find the express charges on a box of merchandise?

4. If you were a freight agent, how would you proceed to find the freight charges on a shipment of furniture?

5. If you know the total number of miles traveled by all the passengers in this country last year and the total number of passengers, how would you proceed to find the average number of miles traveled per passenger?

6. If you know the total number of miles traveled by all the passengers in this country last year and the average number of miles traveled per passenger, what other fact could you ascertain, and how would you do it?

7. If you have a box to send to a certain place, how will you find a practical and economical way to send it, and how will you find the per cent saved by sending it that way instead of a more expensive way?

8. If you know the distance between two stations and the rate of a train per hour, how would you proceed to find the time it takes the train to make the trip?

9. If you know the distance between two stations, the rates of two trains, and the times at which these trains leave the respective stations to travel towards each other on parallel tracks, how do you find the time of meeting?

III. ARITHMETIC OF INDUSTRY

Price List. This price list may be used in solving the problems below:

School workbench . .	$8.75	Plane	$1.85
Chisel, $\frac{1}{4}''$	.25	Vise for woodwork .	8.50
Chisel, $\frac{1}{2}''$	.30	Miter square . . .	.75
Try-square	.30	Steel square . . .	.85

Exercise 23. Buying Tools

1. How much is a 12% discount on a workbench?

2. What is the cost of 7 quarter-inch chisels?

3. Find the cost of 6 half-inch chisels and a plane.

4. Find the cost of a vise, allowing 10% discount.

5. Find the cost of 3 miter squares and 4 steel squares.

6. Find the cost of 8 try-squares, allowing 9% discount.

7. A school buys 6 workbenches, 6 planes, and a vise for woodwork. Find the net price, discount 15%.

Find the cost of the following articles:

8. 6 workbenches, 8 quarter-inch chisels, 6 try-squares, 6 planes, and 1 miter square; discount 8%.

9. 8 half-inch chisels, 6 steel squares, 2 vises for woodwork, 1 miter square, and 4 planes; discount 15%.

10. 6 quarter-inch chisels, 2 planes, 4 try-squares, and 1 steel square; discount 8%.

11. 4 workbenches, 2 quarter-inch chisels, 8 half-inch chisels, 6 planes, 3 vises for woodwork, and 4 steel squares; discount 12%.

Exercise 24. The Cotton Industry

1. A mill bought 8 bales of cotton weighing respectively 496 lb., 510 lb., 524 lb., 495 lb., 503 lb., 498 lb., 503 lb., and 518 lb. If the mill bought on the basis of 500 lb. of cotton to the bale and paid 15¢ a pound, did it gain or lose by the variation in weight, and how much?

2. A manufacturer agreed to take $16\frac{2}{3}\%$ of a cargo of cotton shipped from New Orleans to New York. The cargo consisted of 5460 bales averaging 500 lb. each, and the manufacturer paid 15¢ per pound for the cotton. How many bales did he buy? How many pounds did he buy? How much did the cotton cost him?

3. A certain cotton mill used in one day 5240 lb. of Egyptian cotton costing 33¢ a pound, 14,500 lb. of upland cotton costing 13¢ a pound, and 2274 lb. of Sea Island cotton costing 65¢ a pound. Find the total cost of the cotton used that day.

4. A pound skein of No. 3 cotton yarn contains 2520 yd. How many yards are there in 1525 such skeins?

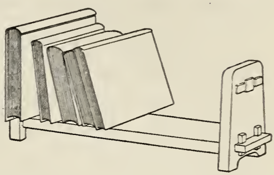
5. A certain manufacturer finds that in cleaning baled cotton he has to allow 30% for waste. If he uses 96 bales of 500 lb. each, how many pounds of waste are there? How many pounds of cleaned cotton are there?

6. In a certain shop the frames for spinning a certain size of cotton thread consist of three sides with 104 spindles to each side, and a certain room has 186 such frames. Find the number of spindles in the room.

7. The 750 employees in a certain cotton mill earn on an average $1.75 a day. If the owners increase the wages of each employee 10%, what is the weekly increase in the total pay roll, allowing 6 da. to the week?

Exercise 25. Wood Work

1. The class made this bookrack. If the rack will hold 16 books $1\frac{1}{2}$ in. thick, how many books $\frac{3}{4}$ in. thick will it hold?



2. The inside length of the shelf is 24 in., each end piece is $1\frac{1}{4}$ in. thick, and the shelf projects $1\frac{1}{4}$ in. at each end. What is the total length?

3. The end pieces are 7 in. high. The shelf is $4\frac{1}{2}$ in. wide, and the greatest width of the end pieces is $5\frac{7}{8}$ in. Taking the total length of the shelf as found in Ex. 2, what length of board 6 in. wide will be needed for the entire shelf, if $\frac{3}{8}$ in. is allowed for waste in sawing and dressing?

4. The price of inch boards of this kind is \$90 per M. At this rate, how much will boards $1\frac{1}{4}$ in. thick cost per M?

5. If a shelf requires a board 48 in. long and 8 in. wide, how many square feet will such a board cover?

6. If a shelf requires a board 48 in. long and 8 in. wide, what will be the cost of the lumber at \$80 per M, if the board is 1 in. thick? if it is $1\frac{1}{4}$ in. thick?

7. If a shelf is $6\frac{1}{2}$ in. wide and 27 in. long, the width is what per cent of the length?

8. If the board in Ex. 7 is $\frac{7}{8}$ in. thick, how many board feet of lumber does it contain? If it is $1\frac{1}{4}$ in. thick, how many board feet does it contain?

9. If a bookcase is 5 ft. high and $6\frac{1}{2}$ ft. long, the height is what per cent of the length?

10. Measure your desk and find how many feet of lumber it contains. What is it worth at \$80 per M?

Exercise 26. The Machine Shop

1. A machinist has a set of drills, the separate drills being marked 0.0625, 0.03125, 0.1250, 0.3125, 0.8750. Express these sizes as common fractions in lowest terms.

2. A foreman of a machine shop showed some boys a casting that was 7 ft. 2 in. long and explained that iron expands $\frac{3}{16}$ in. to the foot when heated to a red heat. Compute the length of this casting when red hot.

3. The foreman told the boys that in casting brass hinges an allowance had to be made for a shrinkage of $\frac{1}{64}$ in length when the brass cooled. Find the length of the mold for a hinge that is to be $5\frac{7}{8}$ in. long.

4. The foreman showed the boys a piece of steel shafting, telling them that it was $2\frac{7}{8}$ in. in diameter, 18 ft. long, and weighed 490 lb. to the cubic foot. He said that he had to know the weight of the shafting in order to be sure to give it ample support. Find the weight of the shafting.

5. The diameter of the piston of the engine in this shop is 10 in. Find the area of the cross section of the piston.

6. If the pressure of steam in a boiler is 140 lb. per square inch, find the pressure on a valve of diameter 3 in.

7. How many R.P.M. (revolutions per minute) must be made by a circular saw with a diameter of 21 in. to give the saw a cutting speed of 6600 ft. per minute?

8. A circular saw making 100 R.P.M. has a cutting speed of 8800 ft. per minute. Find the circumference and the diameter of the saw.

9. The flywheel of an engine is 8 ft. in diameter and makes 250 revolutions per minute. It is directly connected by a belt with a wheel 2 ft. in diameter. How many revolutions per minute does the smaller wheel make?

Exercise 27. The Baking Industry

1. A bakery averages 9000 loaves of bread a day, 6 da. a week, and sells its bread to dealers at 4¢ a loaf. The dealers sell on an average 90% of the bread they get from the bakery, at 5¢ a loaf, and return the balance. How much do the dealers receive per week for the bread?

2. The bakery sells 70% of the bread returned by the dealers at an average price of 3¢ a loaf, and the remainder which is returned each day is ground and sold for $2 the lot. Find the average receipts of the bakery per loaf baked.

3. The bakery spends $1400 a week for flour and $210 for other materials. It pays its workmen $150, and the overhead charges are $145. What is the average cost per loaf?

4. If all the bread were sold at 4¢ a loaf, what would be the per cent of profit?

5. What is the per cent of profit, using the data of Exs. 1, 2, and 3?

6. If flour advances 5% in price and all other charges and the size and quality of the loaf remain unchanged, by what per cent is the baker's profit reduced?

7. If flour is reduced 5% in price and all other charges and the size and quality of the loaf remain unchanged, by what per cent is the baker's profit increased?

8. In making bread the baker uses $\frac{1}{3}$ as much liquid as flour. If he takes 1 pt. of liquid as weighing 1 lb., how many pints of liquid will he use with 40 bbl. of flour?

Allow 196 lb. of flour to the barrel.

9. In Ex. 3 if the wages of the employees are increased 10% and the overhead charges 4%, the other expenses remaining the same, by what per cent is the profit reduced?

Exercise 28. The Machine Industry

1. If a wheel of diameter 2.8 in. is geared with one of diameter 8.4 in., how many revolutions does the smaller wheel make while the larger is making 100 revolutions?

2. Molten brass shrinks $\frac{1}{64}$ of its length in cooling. In casting a brass rod how long must the mold be made in order that the finished rod may be $15\frac{3}{4}$ in. long?

3. To find the weight in pounds of 1 ft. of cast-iron pipe used in pumping, there is a rule for light pipe as follows: Express all dimensions in inches, and multiply 1.08 times the thickness of the metal by 9.82, and this by the sum of the internal diameter and the thickness of the metal. Find the weight of 15 ft. of pipe having an internal diameter of 6 in., the iron being $\frac{3}{8}$ in. thick.

4. In putting in cast-iron pipe where there is a great pressure of water, care has to be taken that a safe pressure is not exceeded. In the machine industry there is a rule for finding the thickness in inches of a $4\frac{1}{4}$-inch pipe that is to stand a pressure of 50 lb. per square inch. Find t, the thickness of this pipe, the formula being:

$$t = \frac{150 \times 4\frac{1}{4}}{7200} + 0.333 \times 0.96.$$

5. In a turbine water wheel the water reaches the wheel through a pipe leading from a considerable elevation. To find the horse power developed by a certain kind of turbine, there is a rule as follows: Multiply the number of cubic feet of water used per minute by the number of feet in the height of the water in the pipe, and divide the product by 660. Find the horse power developed by a turbine water wheel when 520 cu. ft. of water is used per minute and the height of the water is 42 ft.

Exercise 29. Cumulative Review

1. Using short methods, find $37\frac{1}{2}\%$ of \$960; $87\frac{1}{2}\%$ of 248 ft.; $62\frac{1}{2}\%$ of 368 hr.; $83\frac{1}{3}\%$ of \$567.

2. On which will a manufacturer make the greater per cent of profit on the cost, an article which costs \$4.80 to make and sells for \$5.90 less 5%, or one which costs \$7.50 to make and sells for \$9.60 less $\frac{1}{6}$? Prove it.

3. Find the net price of some goods billed at \$1750 less 8%, 4%.

4. A certain locomotive and coal tender together weigh 95 T., and there are 7 cars to their train, averaging 21 T. each. If a horizontal pull equal to 0.5% of the weight of the train is required to maintain the necessary speed on a level track, what is the pull in pounds?

5. Using a short method, divide 74,832 by 125.

6. A car 36 ft. long and 8 ft. 4 in. wide is loaded with wheat to a depth of 4 ft. 6 in. Reckoning a bushel as containing $1\frac{1}{4}$ cu. ft., and a bushel of wheat as weighing 60 lb., how many pounds does the wheat weigh? What is the value of the wheat at \$1.52 per bushel?

7. A manufacturing company has a capital of \$450,000. Its annual business is 3.5 times its capital, and its net profits are 7% of its business. Find the net profits.

8. Express 247 kg. as pounds and 247 lb. as kilograms.

9. A flywheel with a diameter of 6 ft. makes 120 R.P.M. (revolutions per minute). At what rate per hour does a point on the circumference travel?

10. A 12-inch victrola record makes 77 R. P. M. and it requires $4\frac{1}{2}$ min. to play a certain selection. How far does a point on the circumference move in this time?

Exercise 30. Problems without Numbers

1. If you are told the average cost of manufacturing a certain article and the average selling price of the finished article, how can you find the per cent of gain on the cost? on the selling price?

2. If you are told the average cost of selling the article referred to in Ex. 1, how can you find the per cent of gain on the total cost?

3. If a manufacturer raises the wages per day of all his men by a certain per cent and reduces their working day by a certain per cent, what else must you know and how do you find the average increase in wages per hour?

4. What would you need to know and what would you need to do to compute the per cent of net profit on total cost that a manufacturer realizes annually?

5. A dealer finds that he can ship by water a certain per cent cheaper than by rail, but that the additional time required for shipments by water will reduce the market price of his goods by a certain per cent. How can he determine the cheaper plan of shipping?

6. If a manufacturer reduces the wages of his men by a certain per cent and increases the selling price of the finished product by another per cent, assuming that the amount and quality of the work and other factors remain the same, does he increase his net profit by the sum of these per cents? Explain your answer.

7. If a manufacturer increased the wages of his men by a certain per cent and the cost of raw material decreased by the same per cent, would his net profit be the same?

8. Name three ways by which a manufacturer might increase the per cent of his net profits.

IV. ARITHMETIC OF BUILDING

Excavating for a Building. For convenience in working, the excavation for a building is usually extended about 8 in. or 1 ft. beyond the outside of the wall.

To find the number of cubic feet of earth to be removed in making an excavation, we find the product of the length, breadth, and depth, as you have already learned. Since the amount of earth to be removed is usually expressed in loads, we reduce this number of cubic feet to loads, or cubic yards, by dividing by 27.

We may express this by the formula

$$L = \tfrac{1}{27}\,lwd,$$

where L stands for the number of loads, and l, w, d represent the dimensions of the excavation in feet.

Exercise 31. Excavating

1. Given $L = \frac{1}{27}\,lwd$, find the value of L when $l = 24$, $w = 16$, $d = 8$.

2. Given $L = \frac{1}{27}\,lwd$, find l in terms of L, w, and d.

3. An excavation is to be made for a cellar. The dimensions of the outside of the cellar wall are to be 26′ by 30′, but the excavation is to extend 8″ beyond the wall in every direction. The earth is to be removed to a depth of 5′. Find the number of loads to be removed.

4. In Ex. 3 find the cost of the excavation at 54¢ a load, computing to the nearest load.

5. A circular cistern 7 ft. in diameter and 12 ft. deep is to be dug with the excavation extending 18 in. beyond the inside of the wall. Find the cost at 60¢ per cubic yard, computing the excavation to the nearest cubic yard.

Board Measure. We have already learned on page 12 that a board foot (bd. ft.) of lumber means the measure of a piece of lumber 1 sq. ft. on one surface and 1 in. or less in thickness. In speaking of lumber a board foot is usually called simply a foot.

A board 20′ long, 10″ wide, and 1″ or less thick contains $20 \times \frac{10}{12}$ bd. ft., and would sell as 17 bd. ft.

We may express the number of board feet by the formula

$$B = \frac{1}{12}\,lwt,$$

where B is the number of board feet, l the length in feet, w the width in inches, and t the thickness in inches.

Exercise 32. Board Measure

1. Justify the following rules often used in lumber yards for finding the number of feet of lumber 1″ thick: If the board is 4″ wide, take $\frac{1}{3}$ the length; if 6″ wide, take $\frac{1}{2}$ the length; if 8″ wide, take $\frac{1}{3}$ less than the length; if 9″ wide, take $\frac{1}{4}$ less than the length; if 10″ wide, take $\frac{1}{6}$ less than the length; if 12″ wide, take the length.

To take $\frac{1}{3}$ less than the length means to subtract $\frac{1}{3}$ of the length.

2. Lumber dealers frequently use a table, a portion of which is shown below, in determining the number of board feet in a given piece of timber. Complete this table for the sizes 2×10, $2\frac{1}{2} \times 12$, 3×6, 3×8, and 4×6.

Size in Inches	Length in Feet							
	10	12	14	16	18	22	26	30
2×4	$6\frac{2}{3}$	8	$9\frac{1}{3}$	$10\frac{2}{3}$	12	$14\frac{2}{3}$	$17\frac{1}{3}$	20
2×6	10	12	14	16	18	22	26	30
2×8	$13\frac{1}{3}$	16	$18\frac{2}{3}$	$21\frac{1}{3}$	24	$29\frac{1}{3}$	$34\frac{2}{3}$	40

Other Building Materials. In estimating the number of cubic feet of stone or brick required for the walls of a building, outside dimensions are commonly used, the corners being thus counted twice.

The extra labor in laying the corners is usually considered to offset the double counting.

A *perch* of stone is commonly taken as 25 cu. ft., although it is really a little less. The number of perch is therefore found by the formula

$$P = \tfrac{1}{25}\,lwh.$$

In estimating the number of bricks needed for a wall, 22 bricks are usually allowed to 1 cu. ft. The number is therefore found by the formula

$$B = 22\,lwh.$$

Concrete is made of 1 part of Portland cement to 9 parts of gravel, and 1 cu. ft. of cement weighs 150 lb.

Concrete varies, but for our purposes the above is sufficient.

Exercise 33. Excavating and Building

1. A contractor charges 48¢ a load for making an excavation $27' \times 30' \times 6'$ for a cellar. Compute the cost.

2. A cellar floor $24' \times 28'$ is to be concreted at a cost of 64¢ a square yard. Compute the cost.

Call any fraction of a square yard a whole square yard.

3. In Ex. 1 the cellar wall is concrete, its dimensions, measured on the inside, are $24' \times 28' \times 8'$, and it is $18''$ thick. If Portland cement costs \$2 per barrel of 400 lb. and gravel costs 90¢ a load, delivered, how much will the materials for the wall cost?

In all such cases any fraction of a barrel should be counted as a whole barrel.

Lathing and Plastering. Ordinary laths are 4 ft. long, $1\frac{1}{2}$ in. wide, $\frac{1}{4}$ in. thick, and are laid $\frac{1}{2}$ in. apart. They are sold in bunches of 100. The number of bunches (B) of laths required to cover a surface l feet long and w feet wide is given by the formula

$$B = \tfrac{3}{200}\, lw.$$

Contractors usually compute the amount of plastering by the square yard.

The custom varies greatly, and teachers should ascertain the local usage. There is no general rule as to allowance for openings, and in each case this is a matter of special agreement.

Exercise 34. Lathing and Plastering

1. Find the number of bunches of laths needed for a ceiling 30 ft. long and 20 ft. wide; for a wall 16 ft. long and 12 ft. high.

Call any fraction of a bunch of laths a full bunch.

2. If laths cost \$3 per M, nails and labor \$5.50, and plastering 35¢ per square yard, find the cost of lathing and plastering the walls and ceiling of a room 16 ft. × 20 ft. × 9 ft., allowing 15 sq. yd. for openings and baseboard.

3. Find the cost of plastering a schoolroom 22 ft. by 34 ft. by 12 ft., at 34¢ a square yard, making an allowance of 30% of the walls for blackboards, baseboards, and openings but making no allowance for the ceiling.

4. How many laths will be required for the walls and ceiling of a room 15 ft. by 20 ft. by 9 ft., deducting for 2 doors, each 3 ft. by $7\frac{1}{2}$ ft., and 2 windows, each $3\frac{1}{2}$ ft. by 6 ft.?

5. How much will it cost to plaster the room in Ex. 4, at 36¢ a square yard, making full allowance for all openings but no allowance for the baseboard?

Painting and Papering. Painting is usually estimated by the square yard and there is no universal rule as to openings.

The standard width of wall paper is 18 in., and single rolls are 8 yd. long. The width of wall paper varies greatly, especially in high-class and imported papers. Paper hangers usually allow 3 rolls to 100 sq. ft. Parts of rolls cannot be bought, and so a fraction of a roll is counted as a whole roll.

Teachers should ascertain the local usage in regard to openings.

Exercise 35. Painting and Papering

1. The paper running lengthwise and reasonable allowance being made for waste, find the cost of papering a ceiling 15 ft. by 22 ft., at 85¢ a roll, put on.

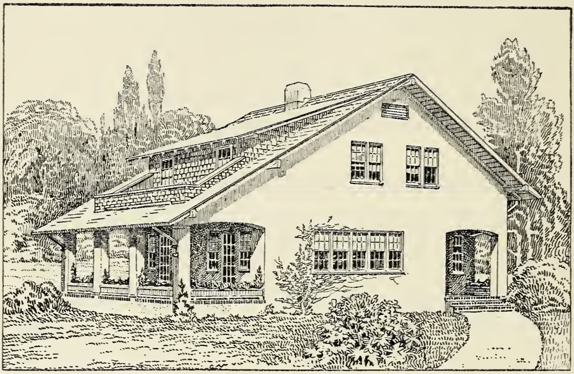
2. How many gallons of paint will be required for 3 coats on a surface 42 ft. long and 20 ft. high, if the first coat requires 1 gal. for 60 sq. yd., the second coat 1 gal. for 70 sq. yd., and the third 1 gal. for 85 sq. yd.?

3. A house 30 ft. by 40 ft. has a double-sloping or A roof and the gables are on the 30-foot sides. The height is 20 ft. to the eaves and 28 ft. to the ridgepole. Find the cost of painting the house at 60¢ a square yard, allowance being made for 3 doors, each 3 ft. by 7 ft. 6 in., and for 16 windows, each 3 ft. by 6 ft.

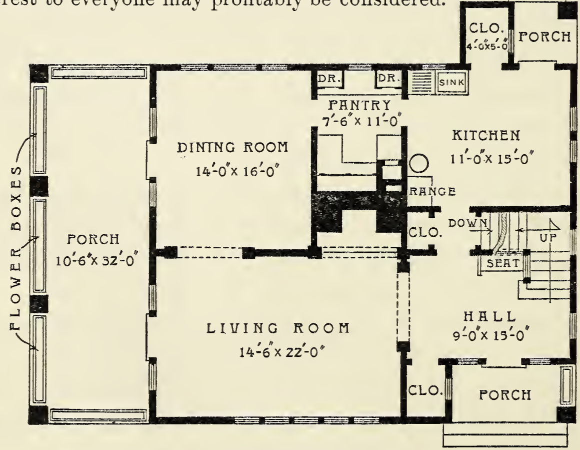
4. The walls of a room are 8 ft. 6 in. high above the baseboard, the perimeter of the room is 76 ft., and 14 ft. is deducted from the perimeter as an allowance for openings. Find the cost of the paper at 32¢ a roll.

5. A room is 16 ft. long, 12 ft. wide, and 10 ft. high. There are 2 doors, each 3 ft. by $7\frac{1}{2}$ ft., and 2 windows, each 3 ft. by 6 ft. How many square yards of plastering are required for the walls and ceiling, the plastering extending to the bottom of the baseboard?

Building a House. Mr. Miller employed an architect to draw plans for a house. Out of several plans, he selected this one, and we shall now compute some of the cost.



The students cannot be expected to enter into the technical details of house building, but a few matters of general interest to everyone may profitably be considered.



FIRST FLOOR PLAN

Exercise 36. Building a House

1. In building the house on page 154 the excavation is made to a depth of 7 ft. under the entire building except under the porch and closet shown in the upper portion of the plan and under the large porch at the left. Compute the cost of the excavation at 56¢ a cubic yard, the excavation extending on every side 1 ft. beyond where the cellar walls are to be built.

2. The walls of the cellar are of concrete 18 in. thick and 8 ft. high, and the floor of the cellar is also of concrete. If the walls cost \$2.85 per cubic yard and the floor costs 60¢ per square yard, compute the total cost of walls and floor of the basement.

3. Deducting from the perimeter of the dining room 25 ft. for the width of the openings, taking the height of the room as 10 ft., and adding 5% for waste, find the cost of the laths for that room at \$3.25 per M.

4. It is estimated that the living room, which is to be heated by hot-water radiators, should have 1 sq. ft. of radiating surface for every 30 cu. ft. of air. At this rate, how many square feet of radiating surface are required if the height of the room is 10 ft.?

5. Find the number of cubic feet of air in the dining room which is 10 ft. high. Allowing 35 cu. ft. of air for every square foot of radiating surface for this room, compute the necessary radiating surface.

6. The total cost of building the house was \$4800 and the lot cost \$1450. The owner computes that he used \$250 worth of his time in planning the house and in overseeing its erection. If he sells the house for \$7280, what is his per cent of profit on the total cost?

Exercise 37. Review Drill

1. Find the single discount equal to each of the following pairs: 20, 10; 10, 10; 20, 20; 20, 5; 40, 40.

2. Express as kilograms: 1400 g., 3248 cg., 8462 mg.

3. Express as centimeters: 0.038 m., 17.2 dm., 1.4 m., $2\frac{1}{2}$ m., 0.00483 km.

4. Taking £1 as equal to $4.87, express in our money: £25, £32.5, £18.5, £124.

5. Taking 1 M. as equal to 23.8¢, express in our money: 35 M., 42 M., 18.5 M., 4270 M.

6. Taking 1 fr. as equal to 19.3¢, express in French money: $34.74, $52.11, $27.02.

7. Write the formula for the volume of a rectangular solid; a cylinder; a cube.

8. Write the formula for the area of a parallelogram; a trapezoid; a triangle; a circle.

9. Write the formula for determining the number of board feet in some lumber.

Perform the following multiplications:

10. 45×75.	**12.** 73×99.	**14.** 75×95.
11. $0.37\frac{1}{2} \times 48$.	**13.** $162\frac{1}{2} \times 328$.	**15.** 424×379.

16. During a fire in a small town the water in the standpipe, which is 28 ft. in diameter, was lowered 7 ft. How many cubic feet of water were drawn from the standpipe?

17. A semicircular arch over a door has a diameter of 4 ft. 2 in. Required the length of the semicircumference.

18. If an iron wheel weighs 16 times as much as the wooden pattern, and the pattern weighs 3 lb. 3 oz., how much does the iron wheel weigh?

Exercise 38. Cumulative Review

1. Using a short method, multiply 1728 by $66\frac{2}{3}$.

2. After increasing its speed $37\frac{1}{2}\%$, an automobile was traveling at the rate of 33 mi. per hour. What was its speed before the increase?

3. Express 748.2 kg. as pounds.

4. A dealer bought a dress for \$45 less 20% and sold it for \$45. What was the per cent of gain on the cost?

5. Express 450 M. as dollars.

6. An agent sold 50 doz. eggs to a grocer at a profit of 15% on the cost. The grocer sold the eggs at retail for \$15.18 and made a profit of 10% on the amount he paid for them. How much did the eggs cost the grocer? How much did they cost the agent?

7. How many quarter-inch tucks can be made in a piece of cloth $6\frac{3}{4}$ in. long, allowing $\frac{1}{4}$ in. between the tucks and $\frac{3}{8}$ in. at each end of the cloth?

8. If corn fodder loses 60% of its weight in curing, how many pounds of green fodder are necessary to produce half a ton of cured fodder?

9. How high is a flagstaff which casts a shadow 80 ft. long at the same time that a post 4 ft. high casts a shadow 6 ft. long?

10. Water is flowing at the rate of 3 cu. ft. in 4 min. into a rectangular tank whose base is 8 ft. 4 in. long and 6 ft. 9 in. wide. How long will it take to fill the tank to a depth of 3 ft. 6 in.?

11. Flint glass contains by weight 72% sand, 14% soda, 12% lime, and 2% aluminum. How many pounds of each of these substances does 750 lb. of flint glass contain?

Exercise 39. Problems without Numbers

1. If you know the number of cylindric pipes in a radiator and the diameter and length of each pipe, how can you find the amount of radiating surface?

2. What must you know and what must you do to find the cost of excavating for a cellar?

3. If you know the dimensions of a brick wall and the cost of bricks per M, how can you find the cost of the bricks in the wall?

4. What must you know and what must you do to find the number of board feet in some oak beams?

5. What must you know and what must you do to find the cost of laths for a room?

6. What must you know and what must you do to find the cost of carpeting a room?

7. If you know the dimensions of a room and of the windows and doors, what else must you know and what must you do to find the cost of plastering the room?

8. If you are told the dimensions of a room and the height and width of all the openings, what else must you know and what must you do to find the cost of laths for the room, allowance being made for all the openings and the height of the baseboard being known?

9. If you are told the surface area of a house which is to be painted and the cost of the paint per gallon, what else must you know and what must you do to find the entire cost of painting the house?

10. What must you know and what must you do to determine the cost of painting a barn, if two coats of paint are to be applied?

V. ARITHMETIC OF BANKING

Depositing Money. Every boy and girl should begin to save money as early in life as possible. Money is usually safer if it is deposited in a bank than if kept at home. Most young people cannot save large amounts, but many banks accept small deposits, and, if made at frequent intervals, small deposits accumulate much more rapidly than one would expect.

The method of depositing money was explained in Book I and is now briefly reviewed.

MERCHANTS BANK
PEORIA, ILLINOIS

Deposited for the account of

..............Robert P. Bell..............

Date..............Sept. 2..............1922

Bills	40	
Gold		
Silver and small coin .	7	50
Check on 1st Nat. Bank	25	
Check on..................Bank		
Total	72	50

The depositor fills out a deposit slip, as shown above.

Exercise 40. Deposit Slips

1. J. P. Ferris deposits \$280 in bills, \$20 in gold, \$3.45 in silver, and checks as follows: Traders Bank, \$48.50; Farmers Bank \$85.60. Write a deposit slip.

2. Suppose that you deposit in some bank to-day \$12 in bills, \$2.40 in silver, and a check on the First National Bank for \$8.50. Write a deposit slip.

3. Write a deposit slip showing the following deposits: Bills, \$275; silver, \$35; checks on Merchants Bank, \$72.80, \$25, \$7.50; check on City Bank, \$223.50.

Bank Book. When the depositor hands the money and the deposit slip to the *receiving teller* he also hands in his bank book, the teller writing the amount on the left-hand page. This book is left at the bank every few weeks to be *balanced*. The following is an extract from a depositor's book, showing deposits and withdrawals:

LEFT-HAND PAGE RIGHT-HAND PAGE

Aug.	1	Balance	2454	48			
	17	M. A. A.	318		VOUCHERS RETURNED AS PER LIST DATED SEPT. 1, 1922	2762	
	19	E. L. M.	25				
	25	E. L. M.	426	75			
	27	W. H. G.	72	50	Balance	534	73
			3296	73		3296	73
Sept.	1	Balance	534	73			

The letters after the dates of deposit are the initials of the names of the receiving tellers. The *vouchers* are the returned checks.

Exercise 41. Depositor's Book

Write two pages like those in a deposit book, using the following items, and balance the account:

1. Oct. 1, balance, $852.68; deposited Oct. 4, $38.50; Oct. 6, $275; Oct. 12, $475.80; Oct. 14, $528.75; Oct. 19, $236.80. Vouchers returned, Nov. 1, $1275.75.

2. Jan. 1, balance, $246.50; deposited Jan. 5, $348.75; Jan. 8, $186.75; Jan. 10, $250; Jan. 14, $426.85; Jan. 24, $342.85. Vouchers returned, Feb. 1, $1462.50.

3. May 1, balance, $375.30; deposited May 3, $62.75; May 4, $286.90; May 12, $425.75; May 16, $526.80; May 18, $48.75. Vouchers returned, June 1, $885.75.

Check. We learned in Book I that when a man having an account in a bank of deposit wishes to pay or draw some money, he writes a *check* for the amount he desires. The following is a common form of check:

> No. *4762* Indianapolis, *Nov. 2,* 19*21*
>
> **Farmers National Bank**
>
> *Pay to the order of*............*P. J. McFarlane*............$*87.*$^{\underline{50}}$
>
> *Eighty-seven* $\frac{50}{100}$~~~~~~~~~~~~~~~~~~~~Dollars
>
> *G. H. Burns*

Various common forms of indorsement are as follows:

P. J. McFarlane	*Pay to the order of M. L. Jones* *P. J. McFarlane*	*For deposit in 1st National Bank* *P. J. McFarlane*
Blank Indorsement	Indorsement to Another Person	Indorsement for Deposit

Exercise 42. Depositing and Drawing Money

1. Assume that you have a deposit of $42.60 in some bank in your city. Write a check drawing out $13.50.

2. S. M. Child has $54.95 on deposit in some bank in your city. Write the check for paying W. R. Brown $20.60.

3. B. R. Watts has $98.50 deposited in a bank. He makes out checks for the following amounts: rent, $28; grocer, $22.60; gas and electric light, $4.40; meat dealer, $7.40. Write each check, inserting names, and find the balance that Watts then has to his credit.

Borrowing Money from a Bank. We found in Book I that business men often have to borrow money from banks, giving their notes, by which they promise to repay the money on demand or in a certain time, usually 30 da., 60 da., or 90 da. The interest on time notes is paid in advance and is called *discount*: the interest on demand notes is paid when the note is paid.

The face of a note less the discount is called the *proceeds*.

The following is a common form of a demand note:

> New York, *May 15*, 192*0*
>
> On demand, for value received, *I* promise to pay to the order of .... *myself* *$1250.00*
>
> *Twelve hundred fifty* $\frac{no}{100}$ ~~~~~~~~ Dollars,
>
> with interest at *2* per cent, at **The Corn Exchange Bank, New York.**
>
> *Richard Cook*

As explained in Book I the forms of these notes vary. City banks frequently lend money to men of high financial standing with no indorsers except themselves. If the sum is very large, or a man's financial standing is not high, the note is made payable to the order of some man of means whose indorsement makes him responsible for the payment; or, as is more commonly the case, the borrower puts some valuable security called *collateral* in the care of the bank, this security being sold if the note is not paid when due.

In the case of the above note the interest cannot be paid in advance because the length of time that the note has to run is not known. Demand notes often draw a low rate of interest, such rates as $1\frac{15}{16}\%$ not being uncommon in large cities. These notes are usually for loans of large sums overnight, to settle stock transactions.

Teachers will find it advisable to discuss pages of this kind informally with the class and to review the forms of time notes studied in Book I.

Exercise 43. Borrowing from a Bank

1. If you borrow \$150 from a bank which charges 6%, what is the interest for 60 da.?

2. A large bank has on deposit an average amount of \$894,600 for a year and pays its depositors an average rate of 2%. It lends \$742,300 at an average rate of $5\frac{7}{8}$%. What is the net interest income for the year?

3. A dealer buys some goods listed at \$1540. If he pays within 30 da. he will be given a discount of 8%. How much will he save if he borrows the money at 6%?

4. If a man lends \$600 at 5% for 18 mo., how much interest will be due at the end of the time?

5. In order to purchase some property a man borrowed \$2500 from a broker, for which he gave a promissory note for 5% for 3 yr. He also borrowed \$2800 from a bank at 6%. Find the total interest for 3 yr.

6. A bank discounts a customer's note on the condition that he is to leave at least 30% of the proceeds as a bank balance. If he discounts a 90-day note for \$2400 at 6%, how much of the proceeds can he use and still meet the bank's requirement?

7. On March 31 the International Motor Co. had a bank balance of \$9700. In order to buy raw material it discounted on April 1 a 60-day note for \$40,000 at 6% and deposited the proceeds. During April and May the company deposited \$18,544 and drew out \$55,450. On May 31 it discounted a new 30-day note for \$30,000 at 6% and deposited the proceeds, at the same time paying off the note given on April 1. During June it deposited \$59,750 and drew out \$12,900. After the note was paid on June 30, what was the balance on deposit?

Discounting Notes. If a dealer buys some goods for the fall trade, but does not wish to pay for them until after the holidays, he may buy them on credit, giving his note. The manufacturer may need the money at once, in which case he will indorse the note and sell it to a bank or a note broker for the face less the discount. Such notes are commonly called *commercial paper.*

For example, if you give a manufacturer your note for \$500, dated Sept. 1 and due Jan. 1, with interest at 5%, and he, needing the money, discounts the note at a bank Sept. 1 at 6%, what are the proceeds?

Face of the note	\$500.
Interest for 4 mo. at 5%	8.33
Amount due at maturity	\$508.33
Discount for 4 mo. at 6%	10.17
Proceeds	\$498.16

The manufacturer may not need the money Sept. 1, and so he may put the note away in his safe and let it lie there drawing interest. But if he needs the money Sept. 16, he may then decide to discount the note, thus:

Face of the note	\$500.
Interest for 4 mo. at 5%	8.33
Amount due at maturity	\$508.33
Discount for 107 da. at 6%	9.07
Proceeds	\$499.26

Banks usually compute the discount period in days. They use tables based on 360 da. to the year, but this is a detail of banking which need not concern the school.

If the banks themselves need more money they may rediscount this paper at the Federal Reserve Bank. The details of the Federal Reserve Bank need not be considered in the school.

Exercise 44. Discounting Notes

1. A 60-day note for \$240 dated May 9 and bearing interest at 5% was discounted May 23 at 6%. Find the discount and the proceeds.

2. Make out a 90-day promissory note for \$360, dated to-day, with interest at $5\frac{1}{2}$%. If this note is discounted 40 da. later at 6%, find the proceeds.

3. Mr. Johnson has a 90-day note for \$420 dated Sept. 8 and bearing interest at $5\frac{1}{2}$%. He discounts the note at 6% at a bank on Oct. 3. Find the discount and the proceeds.

4. A merchant has a 30-day note for \$540 bearing interest at 6% and a 60-day note for \$300 bearing interest at 5%, both notes having run 12 da. He must pay an invoice of \$860 less 5%. If he discounts the two notes at 6% and pays the invoice, how much of the proceeds of the notes will he have left?

5. A real-estate dealer buys 6 lots at an auction at \$780 each. He is required to make a cash payment of at least 20% of the purchase price. He transfers to the owner a 60-day note for \$930 bearing interest at 6%, this note being due 24 da. later. If money is worth 6%, has he complied with the terms of the sale? If not, in what respect has he failed to comply with the terms?

6. Mr. Clark has a 60-day note for \$180 bearing interest at 5%. It has been running for 18 da., and a broker is willing to discount it at 5%, while a bank will discount it at 6%. What would be the difference in the proceeds?

7. Henry Ward holds Charles Smiley's note for 4 mo., dated July 7, for \$450, at $4\frac{1}{2}$%. He discounts the note at 6% on Aug. 15 and deposits the proceeds in a bank where he has a balance of \$257. What is then his bank balance?

8. A merchant has to pay an invoice of \$2700 less 10%. He has \$1350 in the bank and \$375 in his safe. He wishes to leave \$75 in his safe and \$350 in the bank. He also has a 60-day note for \$960 bearing interest at 6%. The note has already run 24 da. If the bank is discounting notes at 6%, how much must the merchant borrow?

As a matter of business the merchant would probably borrow some even amount a little more than this, but for our present purposes the exact amount is sufficient.

9. A man discounted a 90-day note for \$3000 at 6%, used \$2730 of the proceeds, and deposited the balance in the bank where he had a deposit of \$62.50. How much more must he deposit to bring the deposit up to \$2000?

10. A man who had \$1130 balance in a bank needed \$4800 for his business. He could obtain enough by selling some property for \$4000, but instead he discounted at the bank a note for \$4000 for 90 da. at 6%. By holding the property he was able to sell it at an advance of $12\frac{1}{2}$%, and from the proceeds of the sale he paid his note at maturity. What was his net saving by discounting the note?

11. Mr. Phillips buys from the St. Louis Shirt Co. goods amounting to \$1700. He pays half of the bill in cash, and on Sept. 8 he gives his note for 3 mo. for the remainder with interest at 5%. On Sept. 15 the St. Louis Shirt Co. discounts the note at 6%. Find the proceeds.

12. Richard Hodgdon assumed responsibility for two demand notes bearing 5% interest, one for \$4800 dated Jan. 15 and one for \$4600 dated Mar. 24. On Sept. 1 the holders of the notes demanded payment, so that Mr. Hodgdon had to borrow the money. He discounted at a bank his own note for \$10,000 for 60 da. at 6%. How much was left of the proceeds after paying the two notes?

Exercise 45. Cumulative Review

1. A man finds that he can ship his household goods by freight to a certain place for 85¢ per hundred pounds, or for \$58 per carload. If the total weight of his goods when ready for shipment is 12,000 lb., what per cent will he save if he ships the goods as a carload lot?

2. A planer reduces from $2\frac{3}{8}$ in. to $1\frac{11}{16}$ in. the thickness of a board by planing the same amount from each side. How much does he take from each side?

3. A steer weighed 1200 lb., and 46% of its weight was lost in dressing. What was its value when ready for the market, at \$17.50 per hundredweight?

4. Express 250 lb. as kilograms; 250 m. as yards.

5. A jeweler closed out his stock at a reduction of 20%, 30%, the purchaser paying him \$3920. What was the marked price of the stock before the reduction?

6. If a 12-inch gun can fire every 30 sec. a shell weighing 850 lb., how many pounds of shell could a battleship fire from seven 12-inch guns in 2 min. 30 sec.?

7. If 75% of the weight of potatoes is water, and a bushel of potatoes weighs 60 lb., what is the weight of the water in a peck of potatoes?

8. Express as dollars 250 fr.; 300 M; 750 R.

9. If $\frac{3}{4}$ in. on a map represents an actual distance of 38 mi., what distance is represented by $1\frac{7}{8}$ in. on the map?

10. The annual decrease in population in a mining camp for the last 2 yr. has been 5%. If the population is now 1805, what was the population 2 yr. ago?

11. Find the area of a right triangle whose sides are $4\frac{1}{2}$ in., 6 in., and $7\frac{1}{2}$ in.

Exercise 46. Problems without Numbers

1. If you are told the face of a note, the rate of interest which it promises to pay, and the time for which the note was given, what else must you know and what must you do to compute the discount and the proceeds?

2. If you are told the proceeds of a note, the rate of discount, and the period of discount, what else can you find from these values, and how would you proceed?

3. If you are told how much a boy deposits in a bank each month and what rate of interest the bank pays on deposits, what else must you know and what must you do to find how much he will have on deposit at the end of a certain number of years?

4. At the end of a year if you know how much you have withdrawn from your bank account during the year and how much interest the bank has credited to your account, what else must you know and what must you do to compute your balance at the bank?

5. If you discount a note on the day of its date, the note not drawing interest, how do you find the proceeds?

6. If you discount a note on the day of its date, the note drawing interest, how do you find the proceeds?

7. If you discount a note a certain number of days after its date, the note drawing interest, how do you find the discount?

8. In Ex. 7 how do you find the proceeds?

9. If you know your balance in a savings bank at the beginning of the year, the amounts and dates of depositing and of withdrawing money, and the rate of interest allowed by the bank, how do you proceed to find the balance at the end of the first interest period of the year?

VI. ARITHMETIC OF CORPORATIONS

Corporation. The laws of the various states permit a number of persons who wish to go into business together to organize and to act as one body. Such a body is called a *corporation.* A large part of the business of the world is now done by corporations.

The states impose certain conditions upon the establishing of a corporation and certain restrictions upon its acts. For example, there must be a legitimate reason for establishing it, and its acts must be lawful. A corporation is often spoken of as a *company.*

The advantages of *incorporating*, that is, of forming a corporation, are numerous, and a few are seen in the following illustration:

George Adamson has $100,000, of which $40,000 is already invested in business. He wishes to invest the remaining $60,000 in the automobile business, and William Sims has $20,000 which he is willing to invest with him. They believe that it will take $100,000 to establish the business which they have in mind, and so they need $20,000 more. They may now seek for others to go into partnership with them, or they may organize a corporation.

If they organize a partnership, each partner is personally liable for all debts of the concern, and he cannot, in general, sell his interest without the consent of the others. If they organize a corporation, each member (*stockholder*) is liable only to the amount of his *stock*, that is, the amount which he contributes to the corporation, and he may sell his stock without the consent of the others.

George Adamson and William Sims therefore conclude to organize a corporation.

The total amount contributed by the members of a corporation is called the *capital.*

Stock. In organizing a corporation various persons are asked to subscribe for *shares* in the business, or shares of *capital stock.* The subscription list may begin as follows:

THE SUPERIOR AUTOMOBILE COMPANY

We, the undersigned, hereby severally subscribe for the number of shares and the amount of the Capital Stock of the Superior Automobile Company set opposite our respective names, agreeing to pay the said amount on or before September 1, 1920.

Name	Number of Shares	Amount
George Adamson	600	\$60,000

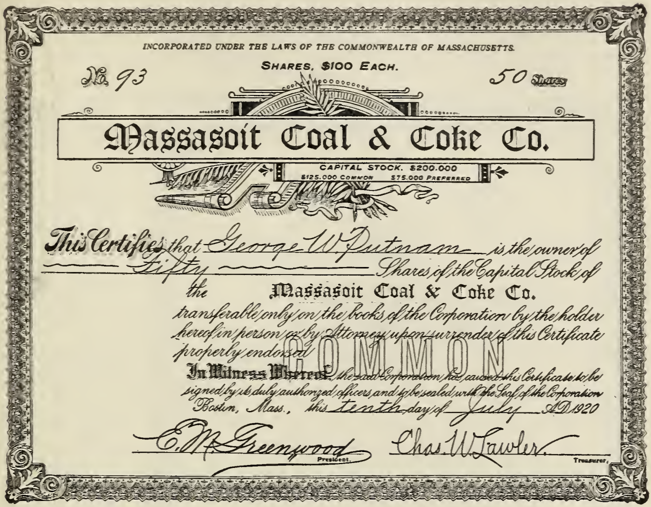
After the stock is subscribed the company *incorporates* according to law, and receives from the state government a document showing that it is a corporation.

Those who subscribed and paid for the stock are now *stockholders* and receive *certificates of stock.* Railway shares are usually \$100 each, although sometimes they are \$50. Mining and other shares are often \$25, \$5, or \$1 each, but the amount varies. In our work the par value of each share is to be taken as \$100 unless otherwise stated.

After all the stock has been subscribed, a meeting of the stockholders is held and a *board of directors* is elected to manage the business. The directors usually elect the *officers* of the company. Each share owned by a stockholder entitles him to one vote. For this reason a few stockholders may control the management of the business.

That portion of the income which is divided among the stockholders makes up the *dividends* of the company.

Certificate of Stock. The following is an example of a certificate of common stock in a corporation:



INCORPORATED UNDER THE LAWS OF THE COMMONWEALTH OF MASSACHUSETTS.

SHARES, $100 EACH.

No. 93 50 Shares

Massasoit Coal & Coke Co.

CAPITAL STOCK. $200.000
$125.000 Common $75.000 Preferred

This Certifies that George W. Putnam is the owner of Fifty Shares of the Capital Stock of the Massasoit Coal & Coke Co. transferable only on the books of the Corporation by the holder hereof in person or by Attorney upon surrender of this Certificate properly endorsed.

COMMON

In Witness Whereof, the said Corporation has caused this Certificate to be signed by its duly authorized officers, and to be sealed with the Seal of the Corporation. Boston, Mass., this tenth day of July A.D. 1920

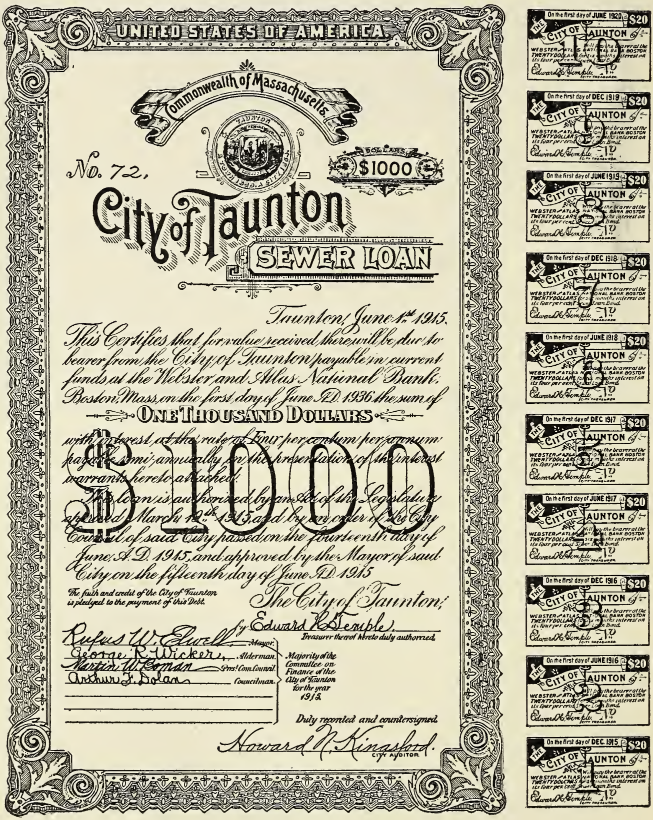
E. M. Greenwood, President. Chas. W. Lawler, Treasurer.

A CERTIFICATE OF STOCK

There are often two kinds of stock: *preferred stock*, which pays a fixed rate of dividend if enough money is earned; and *common stock*, which takes whatever dividends may be paid after other claims have been satisfied.

For example, the preferred stock may guarantee to pay 7% of the amount of the stock, provided that amount has been earned by the company. If the earnings have been such that the common stock regularly receives a dividend of 10%, then the common stock is the more desirable as an investment and will therefore sell for a higher price; but if it receives only 2%, it is less desirable than the preferred stock and will naturally sell for a lower price.

Bond. A written or printed promise to pay a certain sum at a specified time, signed by the maker and bearing his seal, is called a *bond*. When corporations borrow any considerable amount of money, they often issue bonds, agreeing to pay the sum with interest some years later.



UNITED STATES OF AMERICA

Commonwealth of Massachusetts

No. 72.

$1000

City of Taunton

SEWER LOAN

Taunton, June 1st 1915.

This Certifies that for value received there will be due to bearer from the City of Taunton payable in current funds at the Webster and Atlas National Bank, Boston, Mass. on the first day of June A.D. 1936 the sum of

ONE THOUSAND DOLLARS

with interest at the rate of Four per centum per annum payable semi-annually on the presentation of the interest warrants hereto attached.

This loan is authorized by an Act of the Legislature approved March 12th 1915, and by an order of the City Council of said City passed on the fourteenth day of June, A.D. 1915, and approved by the Mayor of said City on the fifteenth day of June A.D. 1915

$1000

The faith and credit of the City of Taunton is pledged to the payment of this Debt.

The City of Taunton;
by Edward H. Temple, Treasurer thereof hereto duly authorized.

Rufus W. Elwell, Mayor.
George R. Wicker, Alderman.
Martin W. Boman, Pres't Com. Council.
Arthur F. Dolan, Councilman.

Majority of the Committee on Finance of the City of Taunton for the year 1915.

Duly recorded and countersigned.

Howard N. Kingsford. City Auditor.

On the first day of JUNE 1920 — $20 — THE CITY OF TAUNTON will pay the bearer at the WEBSTER-ATLAS NATIONAL BANK BOSTON TWENTY DOLLARS for six months interest on its four per cent Sewer Loan Bond. Edward H. Temple, City Treasurer — 10

On the first day of DEC 1919 — $20 — 9

On the first day of JUNE 1919 — $20 — 8

On the first day of DEC 1918 — $20 — 7

On the first day of JUNE 1918 — $20 — 6

On the first day of DEC 1917 — $20 — 5

On the first day of JUNE 1917 — $20 — 4

On the first day of DEC 1916 — $20 — 3

On the first day of JUNE 1916 — $20 — 2

On the first day of DEC. 1915 — $20 — 1

A COUPON BOND

Coupons. Bonds usually have small notes called *coupons* annexed. When the interest is due these are cut off and are collected through a bank.

Mortgage. Bonds of a corporation are usually secured by a *mortgage*, an agreement by which the holders of the bonds may sell the property mortgaged if the bonds and the interest are not paid when due.

The mortgage is usually given to a body of men called *trustees* who act for the bondholders.

Difference between Bonds and Shares of Stock. Bonds differ from shares of stock in this way: the stockholders are the owners of a corporation, while the bondholders are owed money by the corporation. Bonds bear a fixed rate of interest, but the income on shares of stock depends on the net earnings of the company after running expenses and interest on bonds are paid. Stock is like a deed to a farm; bonds are like a mortgage on a farm.

For stocks and bonds as investments see page 223.

Exercise 47. Stocks and Bonds

1. If a company organizes with a capital of \$250,000, how many shares of stock are there at \$100 per share? At \$50 per share how many shares of stock are there?

We shall find later that the *market value* is usually not the same as the face value, that is, the value printed on the face of the certificate and usually called the *par value*.

2. A company borrows \$75,000 for a number of years, in order to improve its plant. If the bonds bear interest at 5%, how much must be set aside each year for interest?

3. A company with capital \$150,000 has \$3750 available for dividends. What rate of dividend can it declare?

4. A company having a capital of \$400,000 declares a dividend of $3\frac{1}{4}\%$. How much money does it thereby distribute among its stockholders?

5. A company declares a dividend of \$80,000, and the stockholders receive \$4 per share. What is the capital?

6. A company with a capital of \$4,000,000 declares a 6% dividend. How much does it distribute in dividends and how much does the holder of 100 shares receive?

7. A company with a capital of \$12,000,000 divides \$1,080,000 in dividends. How much does the holder of 60 shares receive?

8. How much does the holder of 90 shares of railway stock receive when a $5\frac{1}{2}\%$ dividend is declared?

9. A company with a capital of \$500,000 has earned \$30,000 this year above all expenses. It decides to save \$5000 of this for emergencies and to divide the rest in dividends. What is the rate of dividend?

10. A manufacturing company finds that it is necessary to issue 6% bonds to the amount of \$450,000. The capital of the company is \$1,250,000, and it regularly pays a quarterly dividend of 2%. How much must the company lay aside each year for paying the interest and dividends?

Dividends are often paid quarterly, that is, every 3 mo., or semi-annually. Interest on bonds is usually payable every 6 mo.

11. A railway company with a capital of \$75,000,000 and outstanding $4\frac{1}{2}\%$ bonds of \$57,250,000 declared semi-annual dividends last year of $2\frac{1}{2}\%$ each. How much did the company pay in interest and dividends?

In addition to these problems it is desirable that the class should ascertain the facts relating to some local corporation and from these facts make and solve a few problems of the above type.

Exercise 48. Pay Rolls

1. The following portion of a pay roll shows the number of pieces completed by each workman each day of a certain week and the rate or number of cents per piece. It also shows the amounts already advanced in certain cases. Supply the missing numbers in each case.

Name	Number produced						Total	Rate	Earned		Advanced		Due	
	M	T	W	T	F	S								
James Fisher	17	15	18	14	15	8	87	16¢	$13	92	$2	50	$11	42
George Garver	21	18	20	19	19	10		14¢			—	—		
Edgar Hunt	25	20	23	24	27	12		15¢			$3	75		

2. A manager finds that a standard day's work for each workman is 13 pieces, the rate then being 27¢ each. In case of any variation the following *differential rate* is applied:

Number of pieces .	11	12	(13)	14	15	16	17
Cents per piece . .	24	25	(27)	28	30	31	33

The numbers in parentheses indicate the standard. If a man makes less, he is paid less per piece; if more, he is paid more.

The production sheet for a certain week was as follows:

Name	Number produced						Amount due	
	M	T	W	T	F	S		
Robert Hall . . .	12	14	14	17	13	14		
Emmitt Jones . .	14	16	16	17	15	13		

Find the amount due each workman.

Profit Sharing. It is not unusual in large corporations for an employer and his employees to enter into an agreement by which the employees are to receive a share of the profits. In order to participate in the profits it is usually specified that one must have been an employee for a certain period, say at least 6 mo. One large company gives its employees a share of the profits equal to the per cent earned on its common stock. If the common stock earns 8%, an employee whose yearly wages amount to $780 receives 8% of $780, or $62.40, as his share of the profits. Profit-sharing concerns usually allow for a reasonable return (5% to 10%) on the capital invested, after all other expenses have been paid, and then, on the basis of wages or salary, divide the remainder of the earnings among the employees.

Profit sharing is still in the experimental stage and can simply be mentioned as a phase of the arithmetic of corporations.

Exercise 49. Profit Sharing

1. A manufacturing company divides among its employees its annual net profits, after deducting 10% for capital. The net profits are $25,268.40 and the total wages are $280,760. What sum will a workman receive whose annual wages are $936?

2. The gross earnings of a profit-sharing company are $428,080 and the total amount paid out for operating expenses is $334,130. The directors decide to deduct an amount equal to 7% of the capital of $625,000 and to set aside a contingent fund of $25,000, the remainder being divided among the employees on the basis of their annual wages. How much should an employee receive if his annual wages are $1040 and the total annual wages of all the employees are $252,000?

Workmen's Compensation Laws. The laws of most of the states require the payment of some compensation to workmen who happen to be disabled in the course of their employment.

It is desirable that the students should know about the compensation laws of their states. Copies can usually be secured gratis. The details for the various states cannot be given in a textbook of this kind.

It should be explained that, in general, the injury must not result from willful misconduct or the intoxication of a workman.

Corporations usually take out insurance to cover their losses through compensation paid to their workmen under these laws.

Exercise 50. Compensation Laws

1. A large corporation recently estimated that the efficiency of its safety work was distributed as follows:

Organization, 45%, as follows: attitude of officers, 20%; safety committees, 20%; inspection of workmen, 5%.

Education, 30%, as follows: instruction of men, 15%; prizes, 9%; posting signs, 3%; lectures, 3%.

Safeguarding, 25%, as follows: safety devices, 17%; lighting, 5%; cleanliness, 3%.

Represent this statement by a circle graph.

2. A corporation spent $4360 for safety devices and for a safety campaign in its shops. In the next 3 yr. it paid as compensation for injuries only 35% as much as during the preceding 3 yr., when the total payment was $16,240. Allowing interest on the investment for 3 yr. at 6%, compute the saving to the corporation.

3. A corporation finds that a safety campaign in its shops reduced by 48% the number of accidents resulting in lost time. If the number of accidents during the year before the campaign was 350, what was the number during the following year?

Exercise 51. Review of Corporations

1. A manufacturing company buys some crucible plate steel for \$25,300 and some galvanized-iron wire for \$16,500. The company gives its note at a bank for the amount of the bill for 3 mo. at 5% per annum. Find the total cost of the material if the company pays \$262.50 for freight.

2. In a large factory a time detector records, by electricity, the places visited by the watchman in his rounds and the hours. The factory purchased a detector for \$312 and found that the use of the detector reduced its cost for fire insurance by \$80 per year. Find the saving in 5 yr., if money is worth 5%.

3. The National City Real Estate Co. on July 1 collected rent from William Hull, \$365; July 2, from Mrs. Adams, \$87.50; July 5, from Albert Smith, \$175. On July 6 the company sold a lot for \$5200. On July 6 it paid \$138 for clerk hire. Make out and balance a cash account showing receipts from rents and sales, and the disbursements for expenses for the week.

4. The cashier of an importing house started on June 15 with \$785 cash. That day F. M. Case and John Sinclair each paid \$135. He had to pay in American money a bill to Jules Bord amounting to 350 fr. (19.3¢ to a franc) and one to James Coates for £21 (\$4.87 to a pound). Make out and balance a cash account for the day.

5. A production sheet showed that a workman made 28 pieces on Monday, 27 on Tuesday, 29 on Wednesday, 26 on Thursday, 28 on Friday, and 15 on Saturday. The standard was 28 pieces at 12¢ each, or 10¢ each for a less number and 14¢ each for a greater number. The Saturday standard was 15 pieces at 12¢ each. Find the amount due.

Exercise 52. Cumulative Review

1. An invoice for 5 cases of peas @ $1.60 and 25 bbl. of flour @ $9.40, dated Sept. 4, discount 2% 10 da., was settled on Sept. 12. Find the net amount.

2. A jobber offered discounts of 20% and 10%, but the clerk sold an invoice of $350, giving a single discount of 30%. How much did the error cost?

3. By using a certain device a man can run his automobile 14.4 mi. on 1 gal. of gasoline, thus obtaining 20% more mileage than he has obtained without the device. What is his mileage per gallon without the device?

4. If 25% of the students in a certain class failed in an examination and 27 students passed, how many failed?

5. Mr. Wiley having a balance of $428.50 in the bank discounted at 6% a 60-day note for $750 dated Oct. 7. The note bore interest at 5% and was discounted Oct. 24, and the proceeds were added to Mr. Wiley's account. How much was his balance after this was done?

6. In a certain manufactory 8 hours is reckoned a full day of work with 4 hours on Saturdays. Overtime is paid for at the rate of $1\frac{1}{2}$ times the regular rate. On this basis compute the week's wages of each of the following men:

Name	Hours per Day						Time			Overtime		Total Wages
	M	T	W	T	F	S	Hr.	Rate	Amt.	Hr.	Amt.	
George Taylor	8	9	9	8	9	4		28¢				
Albert Turner	9	8	8	8	9	5		32¢				
William Vance	9	9	9	9	9	4		30¢				
James Warner	8	8	9	9	9	5		36¢				

Exercise 53. Problems without Numbers

1. A profit-sharing corporation gives each of its employees a per cent of profit equal to the per cent paid on the common stock. What must you know and what must you do to compute the share of the profit to which a given employee is entitled?

2. What must you know and what must you do to compute the approximate amount of money saved in a certain time by installing safety devices and conducting a safety campaign in a given industrial plant?

3. If you know the annual wages of an employee of a profit-sharing company, the amount of his share of the annual profits, and the annual profits, how can you find the total amount of wages if the profits are distributed on this basis?

4. If you know the capital stock of a company and the amount distributed in dividends for a year, how can you find the rate of dividends?

5. If you know the capital stock of a company and the rate of dividend, how can you find the amount distributed in a certain dividend?

6. If you know the rate of dividends of a company and the amount distributed in dividends, how can you find the amount of capital?

7. If you know the bonded indebtedness of a corporation and the rate of interest paid on the bonds semiannually, how can you find the amount which the company must set aside for interest charges each year?

8. If you know the wages per hour of each workman in a shop, the extra amount paid for overtime, and the number of hours each man works each day of the week, how can you compute the pay roll of the shop for the week?

VII. ARITHMETIC OF HOME LIFE

Division of Income. Those whose duty it is to manage the home should know how to keep an account of family expenses. Good managers do this, and the financial success of the family depends somewhat upon their foresight.

The following table is extensively used. It shows a reasonable division of the annual income for various expenses for a family of five or for a group of four adults:

Income	Food	Rent	House	Clothes	Higher Life
	Per cent	*Per cent*	*Per cent*	*Per cent*	*Per cent*
\$1600–\$4000	25	20	15	20	20
\$1200–\$1600	30	20	10	15	25
\$1000–\$1200	36	20	10	15	19
\$900–\$1000	40	20	10	10	20
\$800–\$900	45	15	10	10	20
\$700–\$800	51	15	10	10	14
\$600–\$700	60	15	5	10	10

This means that a family having an annual income of \$2500 may be expected to spend 25% of it, or \$625, for food; 20%, or \$500, for rent; 15%, or \$375, for operating expenses of the house, such as help, light, fuel, and telephone; 20%, or \$500, for clothes; and 20%, or \$500, for church, travel, lectures, music, savings, and the like.

Exercise 54. Division of Income

Using the above table, find the amount that might reasonably be devoted to food, rent, operating expenses of the house, clothes, and the higher life by families having the following annual incomes:

1. \$675.	**3.** \$825.	**5.** \$1150.	**7.** \$2200.
2. \$750.	**4.** \$950.	**6.** \$1250.	**8.** \$3500.

Household Account. A household account should show a record of items of receipts and payments about as follows:

1. Bank account: deposits, withdrawals.
2. Cash account: receipts, payments.
3. Produce: supplies from farm or garden, at market prices. Enter under receipts if sold; under receipts and also under payments for food if used.
4. Charges of items not paid.
5. Rent. If the property is owned, record the interest on the investment, taxes, insurance, and repairs. If it is rented, include such taxes and repairs as paid by the tenant.
6. Food.
7. Operating expenses: fuel, light, ice, wages, etc.
8. Clothing.
9. Higher life: education, church, charity, amusements, savings, and travel.

The following is one form of a household account:

Dates		Items	Receipts		Payments	
April	1	To Cash on hand	$45	86		
		" Salary	125			
		By Rent			$25	
	3	" Gas			2	60
		" Electric light			1	80
	4	" Grocery bill			5	25
	5	" Milk bill			1	12
	6	" Car fare				20
	7	" Laundry				45
	9	" Balance			134	44
			$170	86	$170	86
	9	Cash on hand	$134	44		

Exercise 55. Household Accounts

Make out a household account similar to the one on page 182 and find the balance for each of the following cases:

1. Cash on hand, May 1, \$39.48. Receipts: allowance for household expenses, \$75. Payments: May 1, grocery bill for April, \$22.18; milk bill, \$3.80; telephone, \$1.25; rent, \$22; May 4, gas bill, \$2.10; electric-light bill, \$1.90; May 11, goods for dress, \$8.50. Balanced May 15.

2. Cash on hand, Sept. 1, \$34.20. Receipts: allowance for household expenses, \$60; interest from investment, \$12. Payments: Sept. 2, Chicago trip, \$18.40; Sept. 5, rent, \$18; Sept. 7, water rent for quarter, \$4.50; Sept. 9, kodak, \$8.50; Sept. 10, gas bill, \$2.10; lecture, \$0.50; Sept. 15, grocery bill, \$18.90. Balanced Sept. 18.

3. Frances, who takes care of her father's house, receives from him \$35 a month, and her sister Mary, who works in a store, gives her \$20 a month and something extra when she can, and Frances does a little work herself. Out of the entire receipts of the family Frances pays the housekeeping expenses and buys the clothes for her brother Louis and herself. Make out the following account for one month and balance it:

Receipts: July 1, father, \$35; Mary, \$20; July 5, helping Mrs. Cole, 75¢; July 8, lace-making, 40¢; July 12, lace-making, 85¢; July 20, trimming hats, \$1; July 27, from Louis, 25¢; July 28, Mary, \$2.

Payments: July 8, butcher, \$3.10; grocer, \$2.15; milkman, 83¢; July 15, butcher, \$4.50; grocer, \$3.80; milkman, 83¢; July 17, house dress, \$2.79; July 19, cap for Louis, 25¢; July 22, butcher, \$3.40; grocer, \$3.85; milkman, 98¢; gas bill, \$2.28; July 29, butcher, \$4.80; grocer, \$4.20; milkman, 73¢; July 31, baker, \$3.20.

Economics of Buying. It is often economical for a housewife to buy supplies in rather large amounts if they are really needed and if there are facilities for storing them safely. The following price list may be used in solving the examples on this page:

Coffee, 35¢ per pound; 5 lb. for $1.60.
Tea, 48¢ per pound; 2 lb. for 90¢.
Maple sirup, 25¢ per pint; $1.45 per gallon.
Honey, 30¢ per bottle; 1 doz. bottles for $3.25.
Buckwheat flour, 6¢ per pound; sack of $24\frac{1}{2}$ lb., $1.30.
Granulated sugar, 8¢ per pound; sack of 25 lb., $1.75.

Students should be encouraged to secure such data from their homes or from stores, making them the basis of practical problems.

Exercise 56. Economics of Buying

1. If a family needs 25 lb. of granulated sugar, what per cent is saved by purchasing it by the sack instead of by the pound?

In such cases reckon the per cent on the higher cost.

2. Several neighbors group together so as to buy groceries more economically. Instead of buying by the pound, they buy coffee in 5-pound lots, buckwheat flour by the sack, and tea in 2-pound packages. What per cent is saved on each of the three articles?

3. What per cent does a woman save in buying maple sirup by the gallon instead of by the pint?

4. What per cent does a housekeeper save in buying honey by the dozen bottles instead of by the bottle?

5. What per cent do you save in buying for $4.50 a laundry book good for $5 worth of work?

6. If ice costs 30¢ per 100 lb., what per cent does a woman save in buying a 1000-pound ice book for $2.50?

Exercise 57. Economics of Management

1. A woman finds that if she hires a dressmaker to make a certain blouse, she must buy $2\frac{3}{4}$ yd. of Persian lawn at 32¢ a yard and $3\frac{3}{4}$ yd. of embroidery at 24¢ a yard, paying $2 to the dressmaker for her work and for incidentals. She can buy a blouse of equally good quality, workmanship, and style for $3.40. Which is the more economical plan and how much more economical?

2. Rolled oats require 1 hr. 45 min. for cooking on a gas range, but if a fireless cooker is used they require only 15 min. on the gas range. If the gas range uses $8\frac{1}{2}$ cu. ft. of gas per hour for this purpose and gas costs $1 per 1000 cu. ft., how much is saved by using the fireless cooker? How much is this saving for a year if rolled oats are prepared 150 times?

3. Rice swells to $3\frac{7}{8}$ times its volume in boiling. In making a pudding by a recipe which calls for 1 qt. of boiled rice, a cook uses 1 pt. of uncooked rice. She uses what per cent more of uncooked rice than the recipe requires?

First find how much rice she should have taken.

4. A housewife finds that rice will boil in 20 min. over a gas burner consuming $8\frac{1}{2}$ cu. ft. of gas per hour, but that she can get the same result by boiling it over the same burner 5 min. and then putting it over the simmering burner 45 min. If the simmering burner consumes only $3\frac{1}{2}$ cu. ft. of gas per hour, which is the cheaper plan? In this way she saves what per cent of the lesser cost?

5. If butter costs 36¢ a pound and butterine 26¢ a pound, and if we allow 2 cups to the pound, how much is saved by using $\frac{2}{3}$ of a cup of butterine instead of butter in making a cake, considering the results as being equally good?

Dietary. Food is cheap or expensive not only with respect to the amount of money it costs but with respect to nutritive material and appetizing qualities.

All foods contain water and all yield energy, and most foods are combinations of five food principles: water, protein, fat, carbohydrates, and ash. A quart of standard milk contains 3.3% protein, 4% fat, 5% carbohydrates, and 0.7% mineral matter (ash).

The following table shows the weights of several foods required to be equal to 1 qt. of milk in nutritive material:

Kind of Food	Total Nutrients Per Cent	Amount required Ounces	Cost per Pound	Net Cost
Beef, round . . .	32.1	13.9	$0.160	$0.14
Oysters	11.7	38.2	.200	.477
Codfish, boneless . .	42.7	10.4	.150	.097
Eggs at 25¢ per doz. .	23.3	19.1	.180	.215
Cheese, Cheddar . .	72.6	6.1	.200	.076
Wheat flour . . .	87.2	5.1	.030	.009
Potatoes at $1 per bu.	17.4	25.7	.016	.025

Exercise 58. Dietary

1. When you buy 1 lb. of beef, how many ounces of nutrients do you secure?

2. Of the above foods, which gives the greatest number of ounces of nutrients per pound? How many ounces?

3. Of the above foods, which is the most expensive, with respect to nutrients, per ounce? How much do its nutrients cost per ounce?

4. How much beef will it take to yield as many ounces of nutrients as are contained in 1 lb. of wheat flour?

Heat Value of Food. Food values are often estimated in terms of their heat value to the body, or in *calories.* Some foods are much richer in calories than others. The government of the United States recently estimated that the number of calories contained in 10¢ worth of various foods is as follows: wheat bread, 2400; cheese, 881; beef, 467; eggs, 198; milk, 736.

The human body needs heat to furnish it with energy, somewhat as an engine needs heat. We obtain this heat and energy partly from our food. If we take into consideration only the calories necessary, we find that wheat bread is one of the best foods.

Exercise 59. Heat Value of Food

1. For the same amount of money, in which of the above foods do we get the greatest number of calories? In which do we get the least? The former is what per cent greater than the latter?

2. The approximate number of calories a day needed by persons of average health is as follows: from 6 to 9 years of age, 80 to 70 calories per kilogram of weight; from 10 to 13 years of age, 70 to 60 calories per kilogram; from 14 to 17 years of age, 60 to 45 calories per kilogram; adults, from 40 to 50 calories per kilogram. About how many calories do you need each day, taking into consideration your weight?

In such scientific tables the metric system is commonly used. The student should find his weight in pounds, reduce this to kilograms (1 kg. = 2.2 lb.), and then solve the problem.

3. If you considered only the largest number of calories needed by you, from which one of the foods mentioned above could you get the required amount for 20¢? How much, to the nearest cent, would the beef cost that would give the required number?

Exercise 60. Ventilation

1. An adult needs 3000 cu. ft. of fresh air per hour. How often does the air of a room $15' \times 18' \times 10'$ need to be changed per hour if the room is occupied by one adult? if occupied by three adults?

2. A lecture hall 14 ft. high contains 450 adults. Allowing 9 sq. ft. of floor space to each, how often should the air be changed to insure good ventilation?

3. A burning gas jet consumes as much air as two adults. A living room is occupied by one adult, and good ventilation is secured by changing the air once in an hour and a quarter during the daytime. How often should the air be changed during the evening if one gas jet is burning? if two gas jets are burning?

4. A kerosene lamp consumes as much air as four adults. When two adults occupy a certain room the air needs to be changed once in an hour and a half during the daytime. When these persons are using a kerosene lamp, how often should the air be changed?

5. In certain hospitals it is the custom to allow 50 cu. ft. of air per minute per person. In a ward $18' \times 60' \times 12'$ there are 12 patients, 2 nurses, and 3 lighted gas jets. How much air should be supplied per hour? How frequently should the air be changed?

See the statements in Ex. 3 for the air consumed by a gas jet.

6. A certain school architect allows 30 cu. ft. of air per minute for students of the junior high school. A classroom contains 30 students and a teacher. The room is $22' \times 30' \times 15'$. How frequently should the air be changed to insure good ventilation?

The question of the schoolroom may profitably be considered.

Exercise 61. Cumulative Review

1. If a sewing machine makes 176 stitches of $\frac{1}{16}$ in. each in a minute, how long will it take to stitch a piece of goods 3 yd. long?

2. A shipper paid \$92.16 freight on some goods at \$0.64 per thousand pounds. How many tons were shipped?

3. A grocer sold 12 boxes of oranges, each box containing 196 oranges, at an average price of $3\frac{1}{2}$¢ an orange. How much did he receive for all?

4. Water in freezing expands 10% in volume. How many cubic feet of ice would 825 cu. ft. of water make?

5. An automobile dealer sold two cars for \$960 each. On the first he lost 20% of what he paid, and on the second he gained 20% of what he paid. How much did he gain or lose on the whole transaction?

6. In casting brass hinges the brass shrinks, in cooling, $\frac{1}{64}$ of its length when melted. What must be the length of a mold for a hinge that is to be $3\frac{5}{16}$ in. long?

7. A cubic inch of iron weighs $4\frac{4}{9}$ oz. What is the weight of an iron bar 1 in. square on the end and $1\frac{1}{2}$ yd. long?

8. A corner lot has 62 ft. frontage and is 150 ft. deep. At 20¢ per square foot, find the cost of laying a walk $4\frac{1}{2}$ ft. wide on the front and side, remembering the corner.

9. A bushel of potatoes weighs 60 lb., and potatoes contain 75% of water. Compute the weight of the water in 15 bu. of potatoes.

10. Each of two office girls receives an increase in wages. The wages of the first are increased from \$8.50 to \$11 a week, and those of the second from \$11 to \$14.50 a week. Compute the per cent of increase in each case.

Exercise 62. Problems without Numbers

1. If you know the cost of a cake of soap and the number of cakes that you can buy for a certain sum, how can you find the per cent saved by buying the larger amount?

2. If you know the amount of cash on hand at the beginning of a month, the amount received during the month, and the expenditures during the month, how can you find the cash on hand at the beginning of the following month?

3. Some neighbors group together to buy potatoes in large amounts. If you know the retail cost per peck and the price at which the purchase is made per bushel, how can you find the per cent saved?

4. If you are told the per cent of nutrients in apples and the price per bushel, what else must you know and what must you do to compare the net cost of nutrients per pound with that of eggs?

5. What must you know and what must you do to find the per cent that a woman saves by purchasing sugar by the large sack instead of by the pound?

6. A man knows his yearly income and his annual expenses for food, rent, operating expenses, clothes, and the higher life. How can he find what per cent of his income is his expenditure for each of these items? How can he find what per cent any item is of each of the others?

7. A woman who is keeping an expense account carelessly enters an expenditure in the receipt column. How will this affect the balance on her books?

8. If you are told the number of pounds of beans that may be purchased for each of two different sums and also the price per pound, how can you determine the economy of buying in the different amounts?

VIII. ARITHMETIC OF FARMING

Farm Accounts. A careful farmer usually keeps a cash book in which he records his receipts and expenditures. No elaborate system is necessary, but a simple record is valuable. The following form is convenient:

Apr.	1	To Balance on hand	148	65		
	3	" 7 doz. eggs @ $0.36	2	52		
	7	By Lumber for shed			29	80
	9	" Groceries			9	40

Such an account should be balanced at the end of the month, or more often if necessary.

The following farm account is kept in more elaborate form, the receipts being placed on the left-hand page and the expenditures on the right-hand page.

Date		Receipts	Dairy		Poultry		Crops		General	
June	1	Milk account	134	80						
	4	9 doz. eggs @ $0.30			2	70				
	5	3 T. hay $17.00					51			

Date		Expenditures	Dairy		Poultry		Crops		House, etc.	
June	1	Cement							12	50
	4	Groceries							5	80
	7	Feed			8	50				
	9	Labor					15	50		
	12	Bran	9	00						

If the size of the book allows, there may be more divisions into columns. Thus, instead of having a column under "House, etc." it would be well to have two or more columns, say for clothing, food, general expenses, etc.

Pages 191–200 may be omitted in city schools if desired.

Inventory. Not only should the farmer keep a cash account and balance it monthly, but he should make an annual *inventory*, that is, a list of all his property and the value at prevailing prices. Machinery should be estimated on the basis of a yearly depreciation of from $4\frac{1}{2}\%$ to 20%, depending on the care given to it and the nature of the machinery. Depreciation should also be allowed on buildings and other property as circumstances require.

The following is the beginning of a farmer's inventory:

List of Property	First Year				Second Year			
	No.	Rate	Valuation		No.	Rate	Valuation	
Real Estate								
Farm and buildings .				23,790				25,740
Cattle								
Dairy cattle .	12	65	780		14	65	910	
Calves . . .	3	7	21		4	7	28	
Two-year-olds	4	9	36	837	3	10	30	968

The following is the summary which the farmer might make for each year at the end of such an inventory:

Property	First Year		Second Year	
Real estate	23,790		25,740	
Live stock	1,872	60	2,064	20
Machinery and tools	540		560	80
Feed and supplies	426	60	436	
Bills receivable	47	40		
Cash in bank	125	60	985	40
Total investment	26,802	20	29,786	40
Less mortgage	4,000		3,200	
Net valuation	22,802	20	26,586	40

Exercise 63. Accounts and Inventories

1. Enter the following according to the first method given on page 191 and balance the account on the latest date: Oct. 9, balance, \$124.30; sold 7 doz. eggs @ 38¢; sold 9 bu. apples @ \$1.10; Oct. 11, paid help, \$14.50; Oct. 15, paid for groceries and meat, \$6.80; Oct. 16, sold a cow for \$85; Oct. 17, sold 4 T. hay @ \$17.50; Oct. 19, paid for harness, \$21; Oct. 20, sold 9 doz. eggs @ 40¢; Oct. 21, sold 5 lb. butter @ 38¢.

2. Enter the following according to the second method given on page 191 and balance the account on the latest date: May 1, balance, \$117.90; May 3, sold 11 doz. eggs @ 34¢; sold 7 lb. butter @ 37¢; May 5, sold milk, \$1.25; May 6, bought separator, \$12.50; paid for groceries, \$3.90; May 12, sold 9 doz. eggs @ 32¢; sold 5 lb. butter @ 35¢; May 13, bought 3 bu. seed potatoes @ \$1.35; May 15, sold wheelbarrow, \$1.75.

3. Enter the following according to the method which seems to you the best for keeping farm accounts: July 1, balance, \$187.50; collected for milk sold to R. M. Arnold, \$7.90; for milk sold to B. W. Spence, \$5.80; July 3, sold 5 doz. eggs @ 28¢; sold 12 lb. butter @ 34¢; July 9, sold 5 T. hay @ \$16; July 11, sold a cow to M. E. Forman for \$92; July 12, paid for labor on crops, \$28.40; paid for groceries, \$12.50; July 15, sold 11 doz. eggs @ 29¢; sold 7 lb. butter @ 35¢; bought chicken feed, \$3.50; July 17, paid for screens for house, \$12.40; July 18, paid for garden seed, \$1.30; July 19, sold 3 doz. eggs @ 28¢. Balance the account July 17.

In rural communities the students may be required to prepare inventories of their fathers' farms or of other property, submitting them to their parents for approval, but not necessarily to the class.

Plant Foods. Growing crops remove certain important plant foods from the soil. The following table shows the number of pounds of nitrogen, phosphoric acid, and potash removed in producing 1 T. of straw or stalks of the crops:

Crops	Nitrogen	Phosphoric Acid	Potash
Corn	16	5	20
Oats	12	5	25
Wheat	10	4	21

Exercise 64. Plant Foods

1. If a 30-acre field averages 2800 lb. of wheat straw to the acre, how many pounds of each of the above plant foods is removed from the soil by the straw in a season?

2. If the straw grown on a 40-acre field of oats removes 720 lb. of nitrogen, how many pounds of straw does the field average per acre?

3. How many pounds of each plant food are necessary to grow a 40-acre field of corn averaging 52 bu. to the acre, each bushel requiring 60 lb. of stalk for its production?

4. When nitrogen is worth 15¢ per pound, phosphoric acid 12¢ per pound, and potash 6¢ per pound, what is the value of the plant food removed from the soil in growing 200 bu. of oats, each bushel requiring 50 lb. of straw?

5. A ton of clover hay contains 41 lb. of nitrogen worth 15¢ per pound, 7 lb. of phosphoric acid worth $12\frac{1}{2}$¢ per pound, and 44 lb. of potash worth 6¢ per pound. If a farmer plows under the crop of a 12-acre field of clover averaging 1.2 T. per acre, what is the value of the plant food returned to the soil?

Cost of Wastefulness. It has been estimated that the average annual depreciation of certain farm machinery and wagons is, to the nearest per cent, as follows:

Machine	%	Machine	%
Corn cultivator . .	7	Hayrake	8
Corn planter . . .	7	Harrow	9
Disk	5	Mower	8
Gasoline engine . .	7	Reaper	8
Grain binder . . .	8	Threshing outfit .	12
Hay loader . . .	12	Wagon	5

Exercise 65. Cost of Wastefulness

1. A farmer paid $80 for a new wagon. He then let it stand out with the result that it depreciated in value 11% the first year. If the average depreciation is as stated in the table, how much did he lose through this lack of care?

2. A man bought a grain binder for $120 and sold it at the end of 3 yr. for $40. Money being worth 6%, find the loss occasioned by lack of care.

Find the average annual depreciation on the following:

3. A corn cultivator costing $32.

4. A 2-horse gasoline engine costing $135.

5. A hay loader costing $75.

6. A harrow costing $15 and a mower costing $40.

7. A threshing outfit costing $2400.

Of course the depreciation does not represent the cost of the machine for a year, for there is also the interest to be considered; but all that is asked for here is the amount of depreciation.

Measuring Land. Farm lands are usually in the shape of rectangles, triangles, or trapezoids, or some combination of these figures. The formulas are as follows:

Rectangle	$A = bh$
Triangle	$A = \frac{1}{2} bh$
Trapezoid	$A = \frac{1}{2} h(b + b')$

Remember that 1 rd. $= 16\frac{1}{2}$ ft. and 1 A. $= 160$ sq. rd.

Exercise 66. Measuring Land

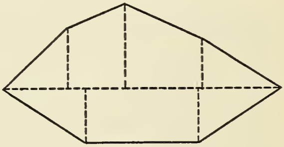
1. Write each of the above formulas as a rule and give an example under each.

2. Find the number of acres in a field 32 rd. by 48 rd.

3. A farmer has two fields to fence, one being 30 rd. by 42 rd. and the other 20 rd. by 52 rd. Find the area of each and the number of rods of fence required for each.

The fact that fields of the same perimeter need not have the same area should be noticed.

4. A farm has the shape shown below. If 1 in. represents 80 rd., compute the approximate area of the field.



Find the approximate lengths of the dotted lines, compute the lengths of the corresponding lines in the field, thus find the areas of the several parts, and then add these areas. In working with such a small figure the results will be only approximately correct, but the work will show one way of measuring such a farm.

5. Find the area of a triangular lot, one side being 72 rd. and its distance from the opposite corner $42\frac{1}{2}$ rd.

Students should, as usual, apply this work to the measurement of the school grounds or of some other convenient piece of land.

Measuring Lumber. You have already learned that the unit for measuring lumber is the *board foot*, usually called simply a *foot*, and that this unit is a piece of lumber 1 ft. long, 1 ft. wide, and 1 in. thick.

In billing lumber, dealers first record the number of pieces, then the thickness and width in inches, and then the length in feet, thus:

$$7 \text{ pc. } 2'' \times 4'' \times 12',$$

this being read "seven two-by-four, twelve feet."

Exercise 67. Measuring Lumber

1. Explain the formula for board feet, $B = \frac{1}{12}lwt$, expressing the formula as a rule and illustrating it.

Find the cost of the following lumber:

2. 9 pc. $2'' \times 6'' \times 16'$, at \$75 per M.

3. 8 pc. $4'' \times 6'' \times 12'$, at \$80 per M.

4. 6 pc. $3'' \times 4'' \times 8'$, at \$70 per M.

5. A convenient rule, known as Doyle's Rule, for measuring the lumber in a log is as follows: Subtract 4 in. from the smallest diameter of the log; multiply the remainder by half itself; multiply the product by the length of the log in feet, and divide by 8. Find the number of board feet in a log 20 ft. long, the smallest diameter being 14 in.

6. Express the rule in Ex. 5 as a formula.

7. A farmer plans to inclose his orchard 22 rd. by 44 rd. by a fence 4 boards high, each board being 12 ft. long, 6 in. wide, and 1 in. thick, this lumber being worth \$27 per M. The posts are to be set 11 ft. apart and cost \$28 per 100. Compute the cost of the lumber for the fence.

Exercise 68. Irrigation

1. An acre-inch of water is the amount of water 1 in. deep that it will take to cover a perfectly flat acre. What is an acre-foot of water, how many acre-inches does it contain, and how many cubic feet of water does it contain?

These terms are used in irrigation, particularly in the mountain states. The exercise may be omitted in those parts of the country in which irrigation is not common. The students should bring local problems to class. The technical terms on this page need not be learned unless needed in the locality, and they are sufficiently explained in the problems to make formal definitions unnecessary.

2. A second-foot of water is a cubic foot flowing past a particular point in 1 sec. If 9 cu. ft. of water pass in 1 sec., the irrigation ditch is delivering 9 second-feet of water. By timing a floating stick in a flume 5 ft. wide, the water being 7 in. deep, it was found that the water was flowing 10 ft. per second. How many second-feet of water was the flume carrying?

Allowance has to be made for friction of flowing water, but this need not be considered in an elementary treatment of the subject.

3. A farmer has 160 A. of beets which he wishes to irrigate from a reservoir. How many cubic feet should he store in the reservoir if each acre requires 2 acre-feet?

There will have to be an allowance for evaporation and other waste, but this problem requires only the net amount of water after such allowance is made.

4. A company furnishes 25 sections (or 25 sq. mi.) of land with $1\frac{1}{4}$ acre-feet of water per acre. If 25% is allowed for loss by seepage and evaporation, so that only 75% of the water is available, and if the reservoir has an average depth of 48 ft. and must store all the water at one time, how many acres will it cover?

Exercise 69. Cumulative Review

1. Express as decimals and as per cents: $\frac{1}{3}$, $\frac{1}{6}$, $\frac{3}{8}$, $\frac{5}{16}$.
2. Divide 360 by 16; by 10; by $0.12\frac{1}{2}$; by $16\frac{2}{3}\%$.
3. Given $A = \frac{1}{2}bh$, find b in terms of A and h.
4. Complete the following bill:

Dayton, Ohio, *May 9*, 19*18*

Mr. *James E. Durrell*

346 W. High St., Richmond, Ind.

Bought of ARNOLD, GREEN & CO.

PIECES	SIZE	LENGTH	BD. FT.	PER M		
25	2″ × 4″	10 ft.		\$32		
62	2″ × 8″	12 ft.		\$34		
18	3″ × 4″	16 ft.		\$35		
12	1″ × 8″	12 ft.		\$31		
				Total		

5. The number of books in the library of a junior high school is being increased at an average rate of $16\frac{2}{3}\%$ each year. There are now 350 volumes in the library. How many volumes did it contain a year ago?

6. In April a coal dealer borrowed \$33,250 at 5% with which to purchase his supply of coal at \$4.75 a ton. He sold the coal at \$5.65 a ton plus an amount such that the purchasers paid for the delivery, storage, and all other overhead charges except interest. He paid his debt after keeping the money 6 mo. How much did he gain?

Exercise 70. Problems without Numbers

1. If you are told the average annual per cent of depreciation in the value of a certain farm machine and the actual per cent of depreciation in the value of the machine, what else must you know and what must you do to find the loss in value due to carelessness?

2. A and B have rectangular farms which require the same number of rods of fence to inclose them. Are the areas of these farms necessarily equal? State your reason and give an illustration.

3. If you are told that all the pieces of lumber in a pile have the same dimensions, and you know the length of each piece, what else must you know and what must you do to find the value of the pile of lumber?

4. If a farmer wishes to build a bin of a certain length and breadth, what else must he know and how can he find how deep it must be built in order that it may hold a given number of bushels of wheat when level full?

5. If you are told the perimeter of a rectangular field and the length of one side, what else must you know and what must you do to compute the cost of a board fence for this field?

6. A farmer finds that if he makes the length and breadth of a wheat bin equal, it will require a certain amount of lumber for a given depth. Will it require more lumber or less lumber, the perimeter and depth remaining the same, if he makes the length greater than the breadth? Why?

7. If a farmer knows the diameter and height of his cylindric silo, how can he find the cubic contents?

8. If the farmer wishes to paint the silo mentioned in Ex. 7, how can he find the area of the surface?

IX. ARITHMETIC OF COMMUNITY LIFE

Nature of Property Insurance. If your home should burn, the loss would be great. It would come suddenly, without any chance of providing for it, and a long time might be needed to save the money required to buy or build a new home. A sudden calamity like this means a great loss to those who have to bear it. For this reason people sometimes join together and contribute each a small sum annually to be used in case of fire. Then if the home of any one of them burns, they pay him enough to fit up a new home from the money they have collected. Thus no one of them suffers any great loss, and everyone feels sure that if a fire consumes his property he will be protected financially as far as such protection may be possible.

This shows the general nature of insurance. Usually, however, instead of a number of persons joining together and paying a little each year, they pay each a small sum annually (a *premium*) to an insurance company, which agrees to pay for some specified loss. The loss against which persons most frequently insure is occasioned by fire, but it may be by other causes.

Insurance. A written agreement to compensate anyone for some specified loss is called *insurance.* The written agreement of an insurance company to pay a certain amount in case of loss is called a *policy.* The sum of money specified to be paid in case of loss is called the *face of the policy.*

The teacher should encourage the students to make inquiry as to the insurance of houses and of various forms of property in the vicinity of the school. In particular, the value of life, accident, and health insurance, of fire insurance, and of stock, crops, boiler, and plate-glass insurance should be discussed.

Premium. The cost of insurance is called the *premium.* The *rate of premium* in fire insurance is usually stated as a specified sum for each \$100 of the face of the policy for a certain period of years, generally from 1 yr. to 3 yr.

What is the premium for insuring a store for \$4000 against loss by fire, at \$1.20 a year?

The premium is $40 \times$ \$1.20, or \$48.

Exercise 71. Property Insurance

Examples 1 to 17, oral

1. What is the premium on a policy for \$800 at 90¢?

2. What is the premium on a policy for \$4000 at 70¢?

3. A man paid \$50 for insuring his house, the rate of premium being \$2. What was the face of the policy?

State the premiums on the following policies at the rates specified:

4. \$2000, \$1.	**6.** \$2750, \$1.	**8.** \$4000, \$1.20.
5. \$1000, 80¢	**7.** \$5200, \$2.	**9.** \$8000, \$2.25.

State the faces of the policies, given the following premiums and rates:

10. \$35, \$3.50. **11.** \$80, \$4. **12.** \$24.25, \$1.

State the rates, given these faces of policies and premiums:

13. \$6000, \$36. **14.** \$7000, \$70. **15.** \$9000, \$180.

16. A building worth \$20,000 is insured for $\frac{3}{4}$ of its value at \$1. What is the premium?

17. A building worth \$9000 is insured for $\frac{2}{3}$ of its value at \$1.10. What is the premium?

18. A building worth V dollars is insured for p/q of its value at r dollars per hundred. What is the premium?

The student has here the same type of example as in Exs. 16 and 17, and he should see that the result gives him a formula that can always be used in cases of this kind.

19. When policies are written for several years the following rates often apply: the rate for 2 yr. is $1\frac{1}{2}$ times the rate for 1 yr.; for 3 yr. it is twice the rate for 1 yr.; for 4 yr., $2\frac{1}{2}$ times the rate for 1 yr.; for 5 yr., 3 times the rate for 1 yr. Mr. Lynch takes out a policy of \$3000 on his house at 80¢ a year and \$1700 on his barn at 90¢. Find the cost of the insurance for 3 yr.; for 4 yr.; for 5 yr.

20. A man insured his factory, valued at \$450,000, for $\frac{4}{5}$ of its value, at \$1.75. What was the premium?

21. A merchant insured his stock, valued at \$16,000, for $\frac{7}{8}$ of its value, at \$1.85. What was the premium?

22. If a three-year policy for \$3500 costs \$52.50, what is the rate of premium for the three years?

23. If a three-year policy for \$6000 costs \$72, what is the rate of premium for the three years? What is the average rate per year?

24. A merchant pays a premium of \$262.50 for insuring his stock for $\frac{3}{4}$ of its value, at \$1.75. What is its value?

25. A manufacturer insures his factory for $\frac{5}{8}$ of its value, at \$2. The premium is \$420. What is the face of the policy? What is the value of the factory?

26. A house is insured for \$10,000 for 3 yr. at twice the annual premium of \$1.40. Find the total premium.

27. If a four-year policy for \$2500 costs \$186.25, and if the four-year rate is $2\frac{1}{2}$ times the annual rate, what is the annual rate per \$100?

Personal Insurance. Insurance against loss due to accident to the insured, his sickness, or his death is called *personal insurance.* The most common form of personal insurance is *life insurance.*

The person to whom personal insurance is to be paid is called the *beneficiary.*

The following are four of the leading kinds of life-insurance policies:

Ordinary Life, the insured paying during his life a certain premium, usually annually or semiannually. Payment is made to the beneficiary on the death of the insured.

The rate is always given as the cost of $1000 worth of insurance; that is, the rate $25.50 means that the annual premium on a policy for $1000 is $25.50.

Limited Life, the premiums being payable for some fixed number of years, after which the policy is called *paid up*, but the face is paid on the death of the insured.

Endowment, the premiums being paid for some fixed number of years, as twenty, ten, or fifteen. At the end of this time many companies give the insured the choice of a certain amount of money in cash, or a paid-up policy payable at his death.

Term Insurance, the premiums being paid for a specified length of time and the face of the policy being payable if the insured dies within the term of insurance.

Thus a man may insure his life or his health for thirty days, as in certain forms of accident insurance. He or his employers may also insure his life for a certain number of years only, as is sometimes done when a man is employed on some important work which might fail if he should die before its completion.

There are various other forms of policies, but most of them are modifications of the above types. Some are arranged so that a certain amount will be paid annually (an *annuity*) during the lifetime of the one for whose benefit they are written.

Exercise 72. Personal Insurance

Examples 1 to 4, oral

1. This table shows the premiums per \$1000 charged by a certain company for various ages:

Age of Insured	Ordinary Life	20-Payment Life	10-Payment Life	10-Year Endowment
20	\$19.21	\$29.39	\$47.85	\$101.57
30	24.38	34.76	56.18	104.14
40	33.01	42.79	67.90	108.07

State a reason for the different rates for the different kinds of policies for a given age.

2. In Ex. 1, state a reason for the different rates for a given kind of policy at different ages.

3. In Ex. 1, state the annual cost of a \$10,000 policy of each kind for a man 30 yr. old.

4. In Ex. 1, state the annual cost of a \$10,000 policy of each kind for a man 40 yr. old.

5. A man took out a \$20,000 policy at \$26.40. He died just before the eighth annual payment was due. How much did his beneficiary receive above what the insured paid to the company? How much would the man have saved in the 7 yr. if he had put the amount of the premium each year at simple interest at 5%?

6. Twenty years ago a man took out a 20-payment policy for \$15,000, paying \$410.85 a year. How much has he paid in the 20 yr.? Estimating that the insurance company has had the use of all this amount for the equivalent of 10 yr., at 5% simple interest, what is the total amount received by the company?

Building and Loan Associations. In many communities institutions known as *building and loan associations* have been organized for the purpose of receiving deposits and of lending money on mortgages.

Depositors make regular monthly payments of a specified amount and receive interest on their balances. The association then lends this money on interest. Each depositor continues in this way until the sum of the monthly payments and the dividends earned equals the face value of his shares. He then receives the face value.

The details of the business of such associations vary so much in different states that students should make inquiry concerning them in case there is an association in their locality.

Exercise 73. Building and Loan Associations

1. A man took 10 shares of stock in a building and loan association on Jan. 10, the monthly payment being \$1 per share. The association credits interest on all payments at the annual rate of 5%, the interest being credited to the account every 6 mo., in this case on July 10 and Jan. 10. If the payments have been made regularly, how much interest was due July 10?

The man receives credit for interest at 5% on \$10 for 6 mo., and similarly for 5 mo., 4 mo., 3 mo., 2 mo., and 1 mo.

2. In Ex. 1, compute the balance to the man's credit at the end of the year, that is, before his second payment on the succeeding Jan. 10, with interest.

As in savings-bank accounts, reckon the interest only on the dollars, not on fractional parts of a dollar. The custom is not uniform, however, in this matter.

3. An association issues 1000 shares on payments of \$1 per month. At the end of each month it loans 70% of the amount paid in. The rate on these loans being 6%, how much interest has been earned at the end of 3 mo.?

Exercise 74. Cumulative Review

1. A farmer builds a cylindric silo of height 26 ft. and diameter 14 ft. What is the capacity in cubic feet?

2. Write a 60-day note for \$150, at 5%, dated to-day, payable to the order of John Doe at some bank. Find the proceeds from discounting the note 10 da. later at 6%.

3. For the age of 21 the annual premium on an ordinary life policy is \$19.12 per thousand. What would be the total amount of premiums paid on \$8000 in 20 yr.? What would be the net amount if the company allowed during this period \$77.80 per thousand in dividends, that is, in shares of the profits returned to the property owner?

4. How much more would the policy of Ex. 3 cost if taken out at the age of 32, the premium being \$25.09, and the dividends for 20 yr. being \$99.65 per thousand?

Using short methods, compute the following:

5. $125 \times \$6400$.

6. $250 \times \$4800$.

7. $\$125{,}000 \div 33\frac{1}{3}$.

8. $66\frac{2}{3}\%$ of \$12,480.

9. In solving a problem a boy multiplies by 3 instead of dividing by 3, and obtains 126 for his answer. There being no other error, what is the correct answer?

10. Make out a cash check for the following sale: $9\frac{1}{2}$ yd. muslin @ 26¢; $3\frac{1}{8}$ yd. velvet @ \$2.60; $12\frac{3}{4}$ yd. lining @ 13¢; $7\frac{1}{4}$ yd. silk @ \$1.80; $4\frac{3}{4}$ yd. suiting @ 84¢; $9\frac{1}{2}$ yd. ribbon @ 36¢. Amount received, \$40.

11. A quart of a certain patent medicine contains $3\frac{1}{2}$ oz. of alcohol. There being 32 fluid ounces in 1 qt., what per cent of alcohol should be indicated on the bottle?

The United States law requires that the per cent of alcohol should be stated in such cases.

Exercise 75. Problems without Numbers

1. If you know the value of a man's house and the fractional part of the value which he wishes to protect by insurance, what else must you know and what must you do to compute the annual premium?

2. If you know the amount of a man's annual premium on a fire-insurance policy and the rate of insurance, how can you find the face of the policy?

3. If you are told what fractional part of the value of a house is insured, the value of the house, and the annual premium, how can you determine the rate?

4. If you are told the face of a policy and the rate, how can you find the premium?

5. If you know how many shares a man has agreed to take in a building and loan association, and the monthly payment on each share, what else must you know and what must you do in order to find the value of the stock at the end of the first year?

6. If you know the annual rate on an ordinary life policy, the face of the policy, and the number of premiums paid before the man died, how would you find the total amount paid to the insurance company?

7. In Ex. 6, how would you find the amount of the interest which the man lost by paying premiums instead of depositing the money in a savings bank?

8. In Exs. 6 and 7, how would you proceed to find the net gain or loss in purchasing the insurance instead of depositing the premiums regularly in the savings bank?

This does not represent the real gain or loss, because the man had the insurance all the time, and even if he had died the day after taking out the policy his beneficiary would have received the money.

X. ARITHMETIC OF CIVIC LIFE

Expenses of Government. Every city, village, town, county, state, and national government must provide money to meet its expenses.

In different states the unit of local government varies somewhat. In some cases the town or village is the important unit, in other cases it is the county. These details need not concern the school unless local conditions demand it.

Cities and villages must pay for fire and police protection, care of streets, lighting, sewerage, erection and maintenance of public buildings, salaries, and the like.

Counties must build roads and bridges, pay salaries, erect certain public buildings, and maintain certain courts and charities.

States must pay the salaries of their officers, maintain certain schools and contribute to certain others, assist in maintaining roads, build prisons and charitable institutions, and perform certain other duties requiring considerable expenditure of money.

The national government of the United States also requires large sums of money to meet its expenses, these expenses averaging over $2,000,000 a day in ordinary years. Among the largest items of national expense are those which relate to preparedness against war, including enormous sums for the navy, the army, and the coast fortifications; to payment for past wars, including many millions for pensions and for interest on our national debt; and to salaries of officials. There are also large expenditures annually for improving our harbors, for erecting such public buildings as post offices and customhouses, and for paying government officers in this country and our diplomatic representatives abroad.

Taxes. The expenses of our national government are met by money which the people are required to contribute in the form of *taxes.*

State and local taxes are usually a certain per cent *levied* on the land, money, and other property of individuals, business concerns, and corporations.

The property to be taxed is valued by officers called *assessors.* The value placed by the assessors upon property for taxation is called the *assessed valuation.*

Upon the assessed valuation a certain *tax rate* is fixed. The words *tax rate* are often used to designate the *number of mills* of tax on each dollar of valuation. Thus, a tax of $5\frac{1}{4}$ mills means $5\frac{1}{4}$ mills on a dollar.

The tax rate is often stated as the number of cents or dollars on \$100, the rate of $52\frac{1}{2}$¢ on \$100 being the same as $5\frac{1}{4}$ mills on \$1.

Finding the Tax Rate. A village with an assessed valuation of \$3,200,000 raises \$16,800. What is the tax rate?

$\$16,800 \div 3,200,000 = \$0.005\frac{1}{4}$.

$$\begin{array}{r} \$0.005\frac{1}{4} \\ 32\cancel{00000})\overline{\$0.168\cancel{00}} \\ \underline{160} \\ 8 \end{array}$$

That is, for every dollar of the assessed valuation of the taxable property that a person living in the village owns, he must pay $5\frac{1}{4}$ mills of tax. Although, as we have seen, the mill is not coined, it is sometimes used for such purposes as this.

Finding the Tax. For example, if a man's property is assessed at \$12,000 and the tax rate is $5\frac{1}{4}$ mills, he must pay $5\frac{1}{4}$ mills on every dollar, or $52\frac{1}{2}$¢ on every \$100; that is, he must pay $12,000 \times 5\frac{1}{4}$ mills or $120 \times 52\frac{1}{2}$¢; or, what is the same thing, he must pay $0.005\frac{1}{4} \times \$12,000$, which is \$63.

Exercise 76. Taxes

1. Mr. Ryan's property is worth \$9000 and is assessed at $\frac{2}{3}$ of its value. The tax rate is 28 mills. What is the amount of his taxes?

2. If the tax rate is 9 mills, what are the taxes of a man whose property is assessed at \$6500?

3. The assessed valuation of a certain village is \$325,000 and the total tax is \$4225. What is the tax rate?

4. A man's tax is \$195 and the rate is 13 mills. What is the assessed valuation of his property?

5. Three men living in different cities were comparing their tax rates. A's rate was \$2.84, B's rate was 28 mills, and C's rate was 2.76%. Which had the largest rate? How much tax should each pay on \$5000?

6. In a certain city property is assessed at $\frac{3}{5}$ of its real value. What is the tax on a house worth \$6500, if the tax rate is 42 mills?

7. If the tax rate is 8 mills, what are the taxes of a man whose property is assessed at \$12,000?

8. If the tax rate is \$2.07, what are the taxes of a man whose property is assessed at \$15,000?

Find the taxes paid on the following, at the rates specified:

9. \$7500, $6\frac{1}{2}$ mills.
10. \$8250, $6.\dot{3}$ mills.
11. \$25,500, \$1.42.
12. \$26,800, \$0.48.
13. \$25,000, \$0.98.
14. \$17,500, \$1.09.
15. \$125,000, \$1.32.
16. \$2,350,000, \$1.52.

17. The assessed valuation of the property in a certain city is \$150,000,000 and the annual city budget amounts to \$1,350,000. What is the tax rate?

Expenses of a City. The authorities of a certain city voted to approve the following budget of expenses:

Police department	\$23,800
Fire department	31,460
Education	87,340
Bureau of charities	4,320
Salaries of city officials	11,170
Street lighting	14,720
Care of streets	6,120
City library	2,000
Health department	3,140
Roads and bridges	9,360
Other expenses	11,070

This budget will be taken as the basis of the problems in the following exercise. Schools will find it profitable to consider the local tax rate, the budget, and the nature of the expenditures.

Exercise 77. Expenses of a City

1. The assessed valuation of the property in the above city was \$17,250,000. What was the city tax rate?

In such a case find the rate to the next highest tenth of a mill.

2. The city was situated in a state where the state tax rate was 3.8 mills and in a county where the rate was 7.2 mills. Find the combined rate of state, county, and city.

3. The next year the item of education was increased 15%, that of roads and bridges decreased 5%, and the rest of the budget increased \$8500. Find the city tax rate.

4. Find the taxes on property assessed at \$7500, using the rates of Exs. 1 and 2.

5. Find the taxes on property assessed at \$4250, using the rate of Ex. 3.

Tax Table. Assessors usually prepare a *tax table* similar to the following. This particular table is arranged for the rate of 17.8 mills on a dollar, or $1.78 on $100, which was the tax rate recently announced in a certain large city.

Tax Table. Rate 17.8 Mills on $1										
	0	1	2	3	4	5	6	7	8	9
0	.000	.0178	.0356	.0534	.0712	.0890	.1068	.1246	.1424	.1602
1	.178	.1958	.2136	.2314	.2492	.2670	.2848	.3026	.3204	.3382
2	.356	.3738	.3916	.4094	.4272	.4450	.4628	.4806	.4984	.5162
3	.534	.5518	.5696	.5874	.6052	.6230	.6408	.6586	.6764	.6942
4	.712	.7298	.7476	.7654	.7832	.8010	.8188	.8366	.8544	.8722
5	.890	.9078	.9256	.9434	.9612	.9790	.9968	1.0146	1.0324	1.0502
6	1.068	1.0858	1.1036	1.1214	1.1392	1.1570	1.1748	1.1926	1.2104	1.2282
7	1.246	1.2638	1.2816	1.2994	1.3172	1.3350	1.3528	1.3706	1.3884	1.4062
8	1.424	1.4418	1.4596	1.4774	1.4952	1.5130	1.5308	1.5486	1.5664	1.5842
9	1.602	1.6198	1.6376	1.6554	1.6732	1.6910	1.7088	1.7266	1.7444	1.7622

Here the first figure of the number of dollars assessed is shown in the column at the left, and the second one is shown in the row at the top. Thus at 17.8 mills on $1 the tax on $100 is $1.78, the tax on $2500 is $44.50, and the tax on $150,000 is $2670.

Illustrative Problems. **1.** How much tax must a man pay on property assessed at $8500, at 17.8 mills on $1?

By the above table the tax on $8500 is $151.30. This is found at the right of 8, the first figure in 8500, and under 5, the second figure in 8500.

2. How much tax must a man pay on property assessed at $17,500, at the rate of 17.8 mills on $1?

By the table the tax on $17,000 is	$302.60
and the tax on $500 is	8.90
Hence the tax on $17,500 is	$311.50

Exercise 78. Taxes

1. Prepare the last two rows of a tax table similar to the one on page 213, the rate being 8.4 mills.

2. From the table prepared in Ex. 1 find the tax on \$8500; on \$9500; on \$86,000.

3. From the table on page 213 find the tax on \$6400; on \$7500; on \$2800; on \$16,750.

4. A corporation owns property assessed at \$4,500,000. If the tax rate is 17.8 mills, what is the amount of the tax paid by the corporation?

5. Find the amount of the various taxes levied on property assessed at \$15,000, the rates being as follows: state tax, $1\frac{1}{4}$ mills; county tax, 2 mills; village tax, 0.9 mill; school tax, 1.8 mills.

6. Some property assessed at \$90,000 was taxed \$1629, and the next year it was assessed at \$94,000 and taxed \$1607.40. Find the tax rate for each year.

7. A man paid \$2200 for a city lot. During the year he paid taxes at the rate of \$0.018 on this amount, and by taking the money out of the savings bank he lost $3\frac{1}{2}\%$ interest. After a year he sold the property for \$2300. Did he gain or lose on the transaction, and how much?

8. A city street half a mile long and 50 ft. wide was repaved at a total cost of \$11,440, the city paying for the intersections of streets and the owners of the property paying half the rest. How much is charged against the owner of a lot with 100 ft. frontage if he is charged for half the cost of paving half the street in front of his lot?

Students should ascertain the tax rate in the place in which they live, the custom as to taxes for paving, the amount raised by taxes, and the nature of the expenditures.

Expenses of the United States Government. The expenses of the United States government vary from year to year, but before the European war they averaged about $2,000,000 a day. Some items of our annual income and expenditures were approximately as follows:

INCOME

Customs (duties on imported goods)	$213,000,000
Internal revenue (tobacco, etc.) . .	388,000,000
Income and corporation taxes . . .	200,000,000
Miscellaneous	58,000,000

EXPENDITURES

War Department	$225,000,000
Navy Department	250,000,000
Pensions	159,000,000
Indians	18,000,000
Interest on public debt	23,000,000
Salaries, diplomatic service, etc. . .	204,000,000

Tariff. The United States collects a large part of its income by a tax on goods brought into the country. This income is called *customs revenue*, *tariff*, or *duty*.

Customs revenue is collected at *customhouses*. These are situated at *ports of entry*.

Goods imported may be on the *free list* and not subject to duty, as raw silk; subject to *ad valorem* (on the value) duty, a certain per cent on the value at the place of purchase, as 15% ad valorem on books; subject to *specific* duty, a certain amount per bushel, or other measure, as 10¢ per bushel of 50 lb. on apples; subject to both *ad valorem* and *specific* duty, as 60% ad valorem and 40¢ per pound on a certain kind of perfumery.

Exercise 79. Tariff

1. The duty on tulip bulbs is \$1 per thousand. What is the duty on an importation of 45,000 bulbs?

2. There is an ad valorem duty of 45% on silk ribbon. Compute the duty on an importation valued at \$1250.

3. The duty on toys is 35%. What is the duty on an importation of toys valued at \$4200?

4. The duty on a certain perfumery is 40¢ a pound and 60% ad valorem. What is the total duty on an importation of 24 lb. of this perfumery valued at \$180?

5. A dealer imports 4000 cigars which weigh $12\frac{1}{2}$ oz. per hundred and are invoiced at \$35 per thousand. The duty is \$4.50 per pound and 25% ad valorem. Compute the duty on the importation.

6. The duty on oriental rugs is 50%. A dealer imports a rug which was valued at \$60 in the country from which it was shipped and computes his expenses of handling the rug as 40% of what it cost him abroad plus the duty. At what price must he sell the rug to make a profit of 20%?

7. When our national income was \$930,000,000 our customs receipts were \$340,000,000. Our customs were, to the nearest tenth per cent, what per cent of our income?

8. During a recent year the total amount received by the government from all import duties was \$213,185,845. The total income of the government was \$779,664,552. What per cent, to the nearest tenth per cent, of the total income was received from import duties?

9. In a recent year our customs revenue was approximately \$213,000,000, which was an increase of 2.4% over the preceding year. Find to the nearest million dollars the revenue for the preceding year.

Internal Revenue. A large proportion of our national revenue comes from the sale of postage stamps and from the taxes on liquors and tobacco, income from the latter source being called *internal revenue.* The receipts of the post-office department are about the same as the expenses.

Exercise 80. Post-office and Internal Revenue

1. In the year in which this country had 55,934 post offices the total receipts from these offices were $312,057,688. Find to the nearest cent the average receipts per office.

2. If the total expenses of the post-office department in the year referred to in Ex. 1 were $306,204,033, find to the nearest cent the average expenses per office.

3. In a year in which the cost of rural free delivery was $50,000,000, the total mileage was 390,000,000. Find to the nearest cent the average cost per mile.

4. In a recent year the government received an income of $159,000,000 from taxes on spirits, 55.3% as much from taxes on tobacco, and 1% more on fermented liquors than on tobacco. Find the total receipts from these sources.

5. In a recent year the internal revenue receipts amounted to $512,700,000. The largest amount paid by any state was 20.46% of this total, which was paid by New York. What was this amount?

6. In a recent year the corporations paid taxes to the government of $56,970,000 and individuals paid taxes of $67,940,000. Find to the nearest tenth per cent the per cent that each is of the sum of the two.

7. Transporting the mails cost $106,900,000 in a recent year, and the total expenditure of the post office was $306,200,000. The former is what per cent of the latter?

Income Tax. Part of the money needed to run the United States government is raised by a tax on incomes. Every citizen of the United States, whether residing at home or abroad, and every person residing in the United States must pay the government 2% of his income each year, $3000 of the income being exempt from taxation. In the case of the head of a family, the income is taxed on the excess above $4000. Persons whose taxable incomes exceed $20,000 a year are taxed more heavily; there is, for example, an extra tax of 1% on incomes in excess of $20,000 and not exceeding $40,000. This extra tax increases in rate as the income increases.

Because of constant changes, only the tax as it was before the European war is here considered.

For example, a married man has an income of $5000 a year. How much is his tax?

He has an exemption of $4000.

He pays 2% on the excess, $1000.

Therefore the tax is 2% of $1000, or $20.

$5000
4000
$1000
0.02
$20

Exercise 81. Income Tax

1. A single man has an income of $3800 a year. What is his income tax?

2. One married man has an income of $4500 a year and another an income of $8500. Find their income taxes.

3. A married man has an income of $35,000. What is his regular income tax of 2%? He pays an extra tax of 1% on the excess of his income above $20,000, after deducting $4000. How much is his total tax?

4. In Ex. 3 what would be the total income tax paid by the man if he were single?

Exercise 82. Cumulative Review

1. The duty on copper ware was recently reduced from 45% to 20%. By how much was the duty decreased on an importation invoiced at $14,800?

2. A dealer buys goods listed at $450 with discounts of 20 and 10. The freight, drayage, overhead, and all other charges amount to $14. The dealer sells the goods for 90% of the list price. What per cent does he gain on the total cost?

3. A dealer bought 400 long tons of coal at $7.40 a ton and sold it at $9.60 per short ton. What was his gain if 8¢ per 100 lb. is allowed for freight, 1% for waste in weight, and $125 for hauling and other expenses?

A long ton is 2240 lb.

4. A boy bought a dozen rabbits for $1.80 and sold them for 30¢ each. What per cent did he gain on the cost?

5. What will it cost to dig a cellar 40 ft. long, 32 ft. wide, and $6\frac{1}{2}$ ft. deep, at 65¢ a cubic yard?

6. A certain map is made to the scale of 40 mi. to 1 in. The map is $7\frac{1}{2}$ in. by 9 in. What is the area represented?

7. A certain field yielded 42 bu. of corn to the acre. The farmer estimated that if he had spent the equivalent of 3 da. more per acre in cultivating the corn, the yield would have been increased 15%. If corn was worth 80¢ per bushel, how much would the farmer have received per acre for his extra work on this basis?

8. Mrs. Arnold's lot has a frontage of 66 ft. on a street 42 ft. wide. If she pays half the cost of paving half the street in front of her property, compute the cost of her paving tax at $2.10 per square yard. If she pays half the cost of the curbing at 62¢ a foot, how much does she pay?

Exercise 83. Problems without Numbers

1. If you know the amount of a man's taxes and the rate of taxation, how can you find the assessed valuation of his property?

2. If you know the assessed valuation of a man's property and the fractional part of the real value represented by the assessed valuation, how can you find the real value?

3. If you are told the real value of some property, the fractional part of the real value which is assessed, and the tax rate for the year, how can you find the amount of the tax on that property?

4. A certain importation is subject to an ad valorem duty of a certain per cent. What must you know and what must you do to compute the duty?

5. If you know the rates of ad valorem and of specific duty that are to be levied upon a certain importation, what else must you know and what must you do to determine the total duty?

6. If you know the total annual income of a single man, how can you compute his income tax?

7. If you know the total annual income of a married man, how can you compute his income tax?

8. If a merchant knows the cost of an article in a foreign country, the per cent of duty that must be paid upon it, and the other expenses incurred in placing the article in stock and selling it, how can he compute the price at which the article must be sold so that he may make a profit of a certain per cent on the total cost?

9. If you know the amount to be raised by taxation in a certain city and the assessed valuation of the property subject to tax, how can you find the tax rate?

XI. ARITHMETIC OF INVESTMENTS

Investments. After a man has saved some money, he will either use it to pay for something he wishes to buy or he will invest it. A man feels much more independent if he has saved money for use in case of need.

The teacher may speak to the class along these lines: Having saved money by wise economy, it is essential that we should know how to take care of it. It is customary to invest money that is not needed for immediate use, so that it will not only be safer than when kept at home but will also bring some income. It is well to remember that the larger the income offered by any form of investment, the greater is likely to be the risk of losing the money put into it. Every boy and every girl should learn something about distinguishing between good investments and bad investments.

Exercise 84. Investments

1. If you have \$85 to invest and deposit it on Jan. 1 in a savings bank that pays 2% interest every 6 mo., how much interest is due you at the end of 6 mo.?

2. If a man has \$2750 to invest and can so invest it as to receive 4% interest, how much income will it bring him each year?

3. A man has deposited \$750 on Jan. 1 in a savings bank that pays 2% interest every 6 mo. How much interest is due him in 6 mo.?

4. A woman has saved \$2250. She can deposit it in a savings bank at $3\frac{1}{2}\%$ a year, or invest it in a mining business which she knows nothing about and which the bankers in her town have never heard of, but which advertises that it will probably pay 16% a year. How much annual income would she get from the bank? Which investment should she make?

Investments in Real Estate. Besides investing in banks and in promissory notes, people who have saved some money often buy real estate; that is, land and buildings.

Real estate makes a particularly desirable form of investment because of its comparative safety, of the tendency of wisely selected property to increase in value, and of the possibility of using such property for a home.

The disadvantages of this form of investment are first, the expenses of maintaining the property, such as insurance, taxes, and repairs, together with the loss of interest on the investment; second, the liability to depreciation in value, which is, of course, generally greater with houses than with land; and third, the difficulty of selling the property readily at its full value.

Let each student in the class select some piece of property and ascertain the price at which it was held five years ago and its present price. What has been the gain or loss per cent during the five years? If it can be ascertained, the amount expended for insurance, taxes, repairs, etc., and the interest on the original investment and on the annual expenses for the five years at 4% or 5% should be deducted from the gain or added to the loss.

Exercise 85. Real Estate

1. Mr. C. H. Johnston paid $6000 for a house. He rents it for $45 a month. He pays $66 a year for taxes, $6 for insurance, and $48 for repairs. What net per cent does he receive on his investment?

2. Mr. Fuller bought a house for $9000 and rented it for $56 a month. He paid annually $100 for taxes, $9 for insurance, and $90 for repairs. At the end of 3 yr. from the time when he purchased the property he sold it for $10,400. How much more did he gain than he would have gained by investing the money for 3 yr. at 6%?

Investments in Stocks and Bonds. Formerly people who had money to invest lent it to borrowers and took their notes, put it in banks, purchased real estate, or became connected with some partnership. At present most great undertakings are carried on by corporations, which issue stocks and bonds as we have already learned, and many people who have money to invest buy these *securities*, as they are called. By purchasing some shares of stock a person becomes a part owner in a railway or in a large business corporation, and by buying a bond he is simply lending his money to some company or government.

Computing Income. Bonds pay a certain rate of interest on the money which they represent; that is, a \$1000 bond which bears interest at 5% pays the owner 5% of \$1000, or \$50, every year; or, as is more commonly the case, such a bond pays \$25 every 6 mo.

If a 5% bond for \$1000 is so much in demand that people are willing to pay more than \$1000 for it, say as much as \$1250, then a holder would get only \$50 a year for what cost him \$1250. That is, his rate of income on the cost would be found as follows:

$$\$50 \div \$1250 = 0.04.$$

Hence he receives 5% on the face of the bond (\$1000), but this is only 4% of what the bond cost him. That is, the income on his investment is 4%.

This does not take into consideration the length of time the bond has to run, for the corporation which issued the bond will pay at maturity only the face value of the bond. The rate of income on a 5% bond for \$1000 bought at \$1250 will, for example, not be 4% if the bond is due in 25 yr., for the holder will receive \$50 interest but will pay \$1250 for what will be worth 25 yr. later only \$1000. These details, however, cannot be fully considered by the school.

Above Par and Below Par. If a stock pays uniformly a high rate of dividend, that is, more than can be received from other safe investments, people will be so anxious to buy it that they will pay more than \$100 for a \$100 share. The stock is then said to be *above par.* If a \$100 share can be bought for just \$100, the dividends are about on a par with other investments, and the stock is said to be *at par.* If the dividends are low the public will usually not care to buy the stock, and it will then be *below par.*

Buying Stock. A purchaser usually buys and sells stock through a *broker*, generally in a place called a *stock exchange*, a kind of auction room for such business.

Brokerage. The broker charges *brokerage* or *commission*, usually $\frac{1}{8}\%$ of the par value of the stock, that is, $12\frac{1}{2}$¢ per \$100 par value of the stock.

Meaning of Stock Quotations. A quotation of $118\frac{3}{4}$ means that a share of stock, if its par value is \$100, will cost the purchaser $\$118.75 + \$\frac{1}{8}$ (or $12\frac{1}{2}$¢) brokerage. Thus the buyer will pay $\$118.87\frac{1}{2}$ per share, while the seller, who must also pay his broker, will receive $\$118.75 - \$0.12\frac{1}{2}$, or $\$118.62\frac{1}{2}$ per share.

In stock quotations fractions are usually expressed in halves, fourths, or eighths, and occasionally in sixteenths. Fractional parts of a share cannot, in general, be bought.

How much must a buyer pay for 10 shares of stock quoted at $137\frac{1}{8}$, and how much will the seller receive, allowing the usual brokerage?

One share costs $\$137\frac{1}{8} + \$\frac{1}{8}$ brokerage, or $\$137\frac{1}{4}$.
Therefore 10 shares cost $10 \times \$137\frac{1}{4}$, or \$1372.50.
The seller, however, receives for each share $\$137\frac{1}{8} - \$\frac{1}{8}$, or \$137.
Hence the seller receives for 10 shares $10 \times \$137$, or \$1370.
That is, each man pays his broker \$1.25.

Exercise 86. Buying and Selling Stock

1. A corporation in which the par value of a share is \$100 declares a dividend of 6%. How much does Mr. Miller receive if he owns 15 shares?

2. A corporation with a capital of \$300,000 has earned \$25,000 this year above all expenses. The directors decide to save \$7000 of this for emergencies and to apportion the remainder as dividends. What is the rate of dividend?

3. A company with a capital stock of \$600,000 apportions \$60,000 in dividends. How much does the holder of 20 shares receive?

As stated on page 170, consider the par value as \$100.

4. Mr. Wilson bought 10 shares of stock quoted at $114\frac{1}{8}$; what did they cost him, allowing the usual brokerage?

5. Mr. Lucas sold 10 shares of stock quoted at $94\frac{7}{8}$; how much did he receive, allowing the usual brokerage?

6. What is the income from 150 shares of stock yielding an annual income of 7% on a par value of \$50?

7. Mr. Thompson bought 30 shares of Baltimore & Ohio stock quoted at $84\frac{1}{4}$ and sold them five days later at $86\frac{1}{2}$. What was his net gain?

In Exs. 7 and 8 the buyer would lose some interest while he held the stock, but again he may receive a dividend during this period. For our purposes, therefore, we shall not consider these items, although an investor would, of course, take each into consideration.

8. George Kent bought 25 shares of American Woolen preferred at $97\frac{3}{8}$ and sold them a week later at $95\frac{1}{4}$. What was his net loss?

9. What is the par value of 80 shares of stock, the par value of a share being \$100? the par value of a share being \$50? the par value of a share being \$10?

Newspaper Quotations. Daily newspapers give the prices of the leading stocks. The problems in the next exercise should be solved by using the following short list or by using the quotations in the daily papers:

At., Top., & S.F.	$105\frac{1}{2}$	Penn. R.R.	$58\frac{1}{4}$
Balt. & Ohio	85	Union Pac.	$148\frac{1}{2}$
N.Y. Central	$108\frac{1}{2}$	Western Un. Tel.	$99\frac{1}{2}$

In the above list the par value of each stock is $100 per share except in the case of the Pennsylvania R.R., where the par value is $50. These differences in par value are technicalities which do not concern the school.

In the following problems relating to stocks or bonds add $\frac{1}{8}$ to the quotation if you are buying, and subtract $\frac{1}{8}$ if you are selling, to pay the brokerage.

If a man buys 100 shares of Western Union Telegraph stock when it is quoted at $99\frac{1}{4}$ and sells it when it is quoted at $98\frac{1}{2}$, how much does he lose on the transaction?

$$\begin{array}{r} \$99\frac{3}{8} \\ \underline{98\frac{3}{8}} \\ \$1 \\ \underline{100} \\ \$100 \end{array}$$

He buys the stock at $99\frac{1}{4} + \frac{1}{8}$ (brokerage), or $99\frac{3}{8}$.
He sells the stock at $98\frac{1}{2} - \frac{1}{8}$ (brokerage), or $98\frac{3}{8}$.
Therefore he loses on a share $\$99\frac{3}{8} - \$98\frac{3}{8}$, or $1.
Hence on 100 shares he loses $100 \times \$1$, or $100.

Exercise 87. Stocks and Bonds

1. Find the cost of 5 M bonds at $105\frac{7}{8}$, the total accumulated interest being $37.80.

Bonds are usually billed to the purchaser as so many M (that is, so many thousand dollars) par value. The purchaser is usually required also to pay the interest which has accumulated since the last interest day, because this will come back to him on the next interest day.

2. Find the cost of 8 M bonds at $103\frac{3}{4}$, the accumulated interest being $6.50 per M.

3. A 5% bond purchased at $96\frac{3}{8}$ yields what rate of income on the investment?

4. A 5% bond is purchased at $124\frac{7}{8}$. What is the rate of income on the money invested?

5. Which gives the better income, 6% stock at $149\frac{7}{8}$ or a $3\frac{1}{2}$% bond bought at $96\frac{7}{8}$?

Compare \$6 ÷ \$150 with \$3.50 ÷ \$97.

6. Disregarding brokerage, which gives the better rate of income, a 5% bond at 120 or a $4\frac{1}{2}$% promissory note?

7. A man buys at $169\frac{7}{8}$ stock that pays 8% dividends. Find the rate of income on the money invested.

8. When United States 4% bonds are at $118\frac{1}{2}$, what rate of income does a purchaser receive on his investment?

Find the gain or loss in buying 50 shares, par value \$100, of the following stocks at the prices quoted on page 226 and selling at the prices quoted below:

9. At., Top., & S.F., $102\frac{1}{8}$.

10. Balt. & Ohio, 86.

11. N.Y. Central, $112\frac{3}{8}$.

12. N.Y. Central, $107\frac{3}{4}$.

13. Union Pac., 146.

14. Western Un. Tel., $99\frac{3}{4}$.

15. Which would you prefer to buy, a stock that regularly pays 8%, quoted at $179\frac{7}{8}$, a 5% bond at $108\frac{3}{8}$, or a $5\frac{1}{2}$% mortgage, the security being equally good?

16. A man buys 100 shares of Peoples Gas & Coke Co. stock at $101\frac{1}{4}$, 200 shares of Southern Pacific at $92\frac{7}{8}$, and 50 shares of U.S. Steel at $107\frac{7}{8}$. Find the total cost.

17. A man buys 100 shares of Northern Pacific at $104\frac{1}{2}$, holds it 5 mo. and sells it at $106\frac{3}{4}$. If money is worth 6% and he has received a dividend of $1\frac{3}{4}$% while holding the stock, how much did he gain or lose on the investment?

Notes and Bonds. Besides bonds of corporations there are bonds given by individuals. For example, a man wishing to buy a farm may have money to pay for only half of it but may think the farm such good property as to be willing to borrow money for the purpose of owning it. An investor may be willing to lend him the money on a note, but usually a bond is given instead.

A note is legal only for a short time, usually 6 yr., after it is due or after the last payment is made on it, but a bond is legal for a longer time. A bond is a longer and more formal document.

To make the bond safe, it is usually accompanied by a *mortgage*, which is practically an agreement that the holder may sell the farm and get his money in case the bond or the interest is not paid. The transaction is usually spoken of as giving a mortgage, although it is really that of giving a bond secured by a mortgage.

The arithmetic which applies to interest on a bond is the same as the arithmetic which applies to interest on a note, and so it is necessary merely to refer to interest on notes as a kind of review.

Exercise 88. Notes and Bonds

1. Find the interest on a bond for \$1750 for 1 yr. at 5%.

2. Find the interest on a note for \$675 for 2 yr. 6 mo. at 6%; for 1 yr. 9 mo. at 5%.

3. A man buys 175 A. of land at \$75, paying two thirds in cash and giving a mortgage for the rest at 6%. How much is the annual interest charge on the mortgage?

This is the usual way of speaking, although really the interest is on the bond which accompanies the mortgage.

4. A man buys 250 A. of land at \$90, paying half in cash and giving a mortgage for the rest at $5\frac{1}{2}\%$. How much is the annual interest charge on the mortgage?

Exercise 89. Cumulative Review

1. A suburban lot was bought for $3600. The taxes in 6 yr. averaged $42.34 a year and there was one special assessment of $93 for repairing the street. At the end of the 6 yr. the lot was sold for $4800. The $3600 could have been invested at 5%. Which would have been the better for the investor, and how much better?

2. A storekeeper borrows $1600 for 60 da. at the rate of 6% per year. He puts the money into his business and makes a profit of 15% on it in the 60 da. He then pays his note, makes another note for $2000 for the same length of time and at the same rate, and again invests the money in his business, but because of bad conditions he loses 2% of this amount. When he pays the $2000 what is his net profit on the two transactions together?

3. A jeweler bought a bill of goods amounting to $3000, terms 30 da. or 2% cash 10 da. He paid cash, borrowing the money for 30 da. at 6%. Before the note was due he sold the jewelry for $3600. What was his profit on this lot of jewelry, not considering overhead charges, etc.?

4. A man invested $5000 in a business that paid 7% annually for 8 yr., after which, owing to hard times, it ceased to pay any profit. He then sold his business for $2400. Had he deposited his money for the 8 yr. in a savings bank paying 4%, withdrawing the interest as due, he would have had his $5000 at the end of the time. How much did he lose by his investment?

5. A private bank agrees to pay 5% interest, compounded annually as in savings banks. How much stands to the credit of a customer who deposits $1600 Jan. 1, 1919, and keeps it there 5 yr., making no withdrawals?

Exercise 90. Problems without Numbers

1. If some mining stock is quoted at a specified amount above par, how do you find the cost of a share?

2. If you know the newspaper quotation on a certain industrial stock, how do you find the amount you would receive for a share?

3. If you know the newspaper quotation on some railroad stock, how do you find the cost of a share?

4. About what would have to be the rate of dividend on a first-class stock, in comparison with the average rate of income on good investments, to have it quoted at par? to have it quoted below par?

5. If you put some money in the postal savings bank, how can you find the amount of the principal and interest at the end of a given time?

6. If you know the number of acres in a farm, the price per acre, the amount which a purchaser pays down, and the rate at which he can borrow money on a mortgage to pay the balance, how will you find the annual interest charge on the property?

7. If an investor purchases a certain number of shares of stock through a broker, how can you find the amount of the brokerage?

8. If an investor buys a certain number of bonds, the par value and rate of interest on the bonds being known, the interest to be paid semiannually, how can you find the semiannual interest?

9. In Ex. 8, if the investor buys the bonds at a certain per cent above par, how can you find the rate of income on his investment, not considering the date of maturity of the bonds?

XII. ARITHMETIC OF MENSURATION

Nature of the Work. In Book I you studied the geometry of size, and in studying algebra in this book you applied the formula to the areas and volumes of certain figures. This part of mathematics is often called *mensuration.* You learned how to measure the surfaces commonly met and how to find the volumes of many solids. We shall now review this work, and shall consider a few more solids which are often found in the arithmetic of industry.

Exercise 91. Review of Surfaces

1. Find the area of a floor 12 ft. 6 in. by 17 ft. 6 in.

2. Find the area of a triangle if the base is 9 ft. 4 in. and the height 6 ft. 8 in.

3. Find the area of a parallelogram of which the base is 4 ft. 8 in. and the height 2 ft. 6 in.

4. Find the area of a trapezoid that has the two bases 6 ft. 4 in. and 4 ft. 6 in. and the height 2 ft. 3 in.

5. Find the area of a circle of radius 4 in.

6. Find the area of a circle of diameter 7 in.

7. Find the area of a circle of circumference 22 in.

8. The circumference of a cylindric column is 44 in. Find the diameter and the area of the cross section.

9. In Ex. 8, if the height is 20 in. find the area of the curved surface.

10. Find the area of the curved surface of a cylinder $3\frac{1}{2}$ in. high and $3\frac{1}{2}$ in. in circumference.

11. In Ex. 10 suppose the height to be 2 ft. 7 in. and the circumference 4 ft. 8 in.

Exercise 92. Review of Volumes

1. Find the number of cubic inches in a box 1 ft. 8 in. long, 9 in. wide, and 1 ft. 1 in. deep.

2. Find the number of cubic inches in a cylindric jar of diameter 9 in. and height 1 ft. 2 in.

Given that 1 gal. contains 231 cu. in., and that 1 bbl. contains $31\frac{1}{2}$ gal., solve Exs. 3–6:

3. What is the capacity in gallons of an aquarium 16 in. long, 10 in. wide, and 1 ft. deep?

4. A tank is 8 ft. 6 in. deep and the bottom is 6 ft. 4 in. square. What is the capacity in gallons?

5. In Ex. 4 what is the capacity in barrels?

6. A tank is 9 ft. 3 in. long, 6 ft. 6 in. wide, and 5 ft. deep. What is the capacity in barrels?

7. How many tons of coal, 35 cu. ft. to a ton, can be stored in a bin 16 ft. long, 8 ft. 6 in. wide, and 5 ft. 6 in. deep?

8. A dealer stores coal to an average depth of 5 ft. 4 in. in an inclosure 75 ft. long and 40 ft. wide. At 35 cu. ft. to a ton, how many tons of coal are stored in the bin?

9. A brick wall is 128 ft. 6 in. long, 1 ft. 6 in. thick, and 4 ft. 4 in. high. Allowing 22 bricks to 1 cu. ft., find to the next greater hundred the number of bricks used.

10. A certain kind of fruit is shipped in boxes 9 in. by $15\frac{1}{4}$ in. by 9 in. Find the capacity in cubic inches.

11. How many cubic feet are there in a wall 16 ft. long, 6 ft. 8 in. high, and 14 in. thick?

12. In making a concrete wall 18 ft. 3 in. long and 9 ft. 4 in. high, the contractor used $255\frac{1}{2}$ cu. ft. of concrete. Find the thickness of the wall.

13. A tank with base 10 ft. by 18 ft. contains 900 cu. ft. How deep is the tank?

14. A box with base 15 in. by 18 in. contains 4500 cu. in. How deep is the box?

15. A tank 5 ft. deep contains 680 cu. ft. What is the area of the base?

16. A hall is 18 ft. high and 40 ft. long and has a capacity of 28,800 cu. ft. Find the width of the hall.

17. A block containing 900 cu. ft. has a base area of 40 sq. ft. How high is the block?

18. A hall contains 37,583 cu. ft. It is 41.3 ft. long and 36.4 ft. wide. How high is the hall?

19. A box contains 157.8 cu. in. The area of the bottom is 52.6 sq. in. How deep is the box?

20. A block of granite is 7 in. long, 4.8 in. wide, and contains 168 cu. in. How thick is the block?

21. The floor of a hall contains 2772 sq. ft., and the hall contains 30,492 cu. ft. Find the height of the hall.

22. A storeroom is 28 ft. long, 27 ft. 6 in. wide, and has a capacity of 7700 cu. ft. How high is the room?

23. A bedroom contains 1368 cu. ft. If the floor is square and contains 144 sq. ft., find the three dimensions.

24. A room containing 2537.5 cu. ft. is 17.5 ft. long, and the area of the floor is 253.75 sq. ft. What are the three dimensions of the room?

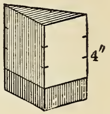
25. A room contains 3600 cu. ft. The floor is square and contains 400 sq. ft. Find the dimensions of the room.

26. Find the edge of a cube that contains 3375 cu. in.

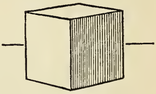
Find by trial the three equal factors of 3375.

Prism. A solid having two parallel faces exactly alike and all the rest of the faces parallelograms is called a *prism.*

The solid shown in the figure is a prism, the parallel faces in this case being triangles. The *bases* of a prism are the two parallel faces, and the *height* or *altitude* of the prism is the distance between these bases. If all the faces of a prism are rectangles, the prism is called a *rectangular prism.* An ordinary crayon box and a brick are common examples of rectangular prisms.



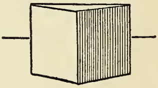
Volume of a Prism. Contractors, builders, and mechanics frequently have to find the volume of a prism of some kind. For example, a contractor computing the amount of earth to be removed in excavating a ditch uses the rule for the volume of a prism. To find this rule first consider the following questions:



If this kind of prism, a cube, is 1 in. on an edge, what is its volume?

If a cube is 2 in. on an edge, what is its volume? If a cube is 3 in. on an edge, what is its volume?

This figure represents half of the above cube. If the cube is 1 in. on an edge, what is the volume of this half?



If the cube is 4 in. on an edge, what is the volume of this half?

If the area of the base of the first figure on this page is 5 sq. in., what is the volume of the lower shaded part, which is 1 in. high? What is the total volume?

The volume of any prism is found in the same way as that of a triangular prism.

The volume of a prism is equal to the area of the base multiplied by the height.

Expressing this as a formula, we have

$$V = bh.$$

Exercise 93. Prisms

Find the volumes of prisms with the following bases and altitudes:

1. 428 sq. in., 26 in.

2. 48.8 sq. in., 6.3 in.

3. 17.46 sq. ft., 3.1 in.

4. 63.6 sq. ft., 3.2 ft.

Find the altitudes of prisms with the following volumes and bases:

5. 530.1 cu. in., 31 sq. in.

6. 386.1 cu. ft., 28.6 sq. ft.

7. 2385.79 cu. yd., 94.3 sq. yd.

8. 8309.02 cu. ft., 428.3 sq. ft.

9. A rectangular granite block is $5\frac{1}{2}$ ft. high and has a base $2\frac{1}{2}$ ft. by $7\frac{1}{2}$ ft. Find the cost of polishing the sides and top of the block at 6¢ per square foot.

10. Find the weight of the air in your schoolroom, allowing 12 cu. ft. of air to the pound.

11. A rectangular stone is $3\frac{1}{2}$ ft. by $4\frac{1}{2}$ ft. by 5 in. Find the weight of this stone at 154 lb. to the cubic foot.

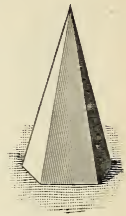
12. A granite block is $7\frac{1}{2}$ ft. high. The base is a right triangle and the sides which form the right angle are 14 in. and 10.5 in. respectively. Find the cost of polishing the sides and top of the block at 54¢ per square yard.

13. A farmer wishes to build a bin that will hold 960 bu. of wheat. If the base of the bin is to be 18 ft. by 16 ft., how deep should it be?

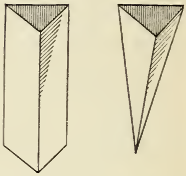
Allow $1\frac{1}{4}$ cu. ft. to a bushel, and take the bin as level full.

14. The base of a certain prism is a triangle whose base is 14 in. and whose height is $7\frac{1}{2}$ in. The height of the prism is 18 in. Find the volume.

Pyramid. A solid of this shape, in which the *base* is any polygon and the other faces are triangles meeting at a point, is called a *pyramid.* The point at which the triangular faces meet is called the *vertex* of the pyramid. The distance from the vertex to the base is called the *height* or the *altitude* of the pyramid.



Volume of a Pyramid. A contractor or builder occasionally needs to find the volume of a pyramid. The rule for finding the volume is easily verified by taking a hollow prism and a hollow pyramid of the same base and the same height, as here shown, filling the pyramid with water and pouring the water into the prism. It will be found that the prism can be exactly filled with three times the amount of water that fills the pyramid.



Therefore the formula for the volume is

$$V = \frac{1}{3} bh.$$

Exercise 94. Pyramids

1. If you know the volume and height of a pyramid, how can you find the area of the base? State the formula.

Find the volumes of pyramids with these bases and heights:

2. 423.6 sq. in., 4.6 in.

3. 105.3 sq. in., 6.2 in.

4. 576.8 sq. ft., 13.5 ft.

5. 482.1 sq. yd., 11.1 yd.

Find the bases of pyramids with these volumes and heights:

6. 89.6 cu. in., 12.8 in.

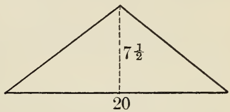
7. 140.4 cu. in., 23.4 in.

Exercise 95. Cumulative Review

Using short methods, find the values of:

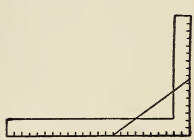
1. $125 \times \$1680$.
2. $1\frac{1}{4} \times 12{,}480$.
3. $33\frac{1}{3} \times 48{,}270$.
4. $66\frac{2}{3} \times 48.375$.
5. $37\frac{1}{2} \times 129.848$.
6. $999 \times 112{,}223$.

7. A carpenter is constructing the roof here shown, the building being 20 ft. wide and the gable $7\frac{1}{2}$ ft. high, and he wishes to know the length of the slope. He draws a plan to the scale $\frac{1}{30}$. How many inches on the plan will represent the $7\frac{1}{2}$ ft., or "rise," as the carpenter calls it?



8. The carpenter finds that the plan of Ex. 7 is not convenient, so he draws a right triangle showing half the plan, and represents by a line 10 in. long the 10 ft., or "run," as he calls it. Draw such a plan and measure the slope. What does the carpenter find to be the length of the slope of the roof?

9. In practice the carpenter would proceed in another way. He would take his square, and on the tongue, or short arm, he would take a point $7\frac{1}{2}$ in. from the corner; then on the blade, or long arm, he would take a point 10 in. from the corner. He would then measure the distance between these points, and the number of inches would be the same as the number of feet in the slope of the roof. Draw the figure and show why this plan gives the correct result.



Find the heights of pyramids with these volumes and bases:

10. 871 cu. in., 174.2 sq. in.
11. 3570.6 cu. ft., 324.6 sq. in.

Exercise 96. Problems without Numbers

1. How do you find the square of a number?

2. If you know the side of a square, how do you find the area of the square? Illustrate by an example.

3. If you know the area of a square, how do you find the side of the square? Illustrate by an example.

4. How do you find by factoring the square root of a number that is a perfect square? Illustrate.

5. If you know three terms of a proportion, how do you find the fourth term?

6. How do you extract the square root of an integer that is a perfect square?

7. How do you extract the square root of a common fraction? How do you extract the square root of a decimal or of a number that contains a decimal?

8. How do you find the hypotenuse of a right triangle when you know the other two sides?

9. Suppose that a fly starts from the upper northeast corner of a room and flies in a straight line to the lower southwest corner. How can you find the distance he flies, knowing the three dimensions of the room?

10. How would you find the number of cubic feet in a cylindric water tank?

11. If you know the base and altitude of a pyramid, what else can you find, and how do you find it?

12. If you know the altitude of a cylinder and the diameter of the base, what other two things can you find, and how do you find them?

13. If you know the volume and the area of the base of a pyramid, how can you find the height? State the formula.

MATERIAL FOR DAILY DRILL

EXERCISE 1

Taking (a) *5.4*, (b) *6.8*, (c) *11.6*, (d) *13.8*, (e) *14.6*, *or* (f) *16.2*, *as the teacher directs:*

1. Add it to $0.936 + 27.004 + 148 + 81.07 + 9.68$.

2. Subtract it from $82.738 + 9.262$.

3. Multiply it by itself, and multiply the product by $2\frac{1}{4}$.

4. Divide it by 0.23, carrying the quotient to the nearest hundredth in this case and in all similar cases.

5. Find 4.3% of it.

EXERCISE 2

Taking (a) *285.4*, (b) *337.8*, (c) *346.82*, (d) *365.36*, (e) *392.18*, *or* (f) *394.124*, *as the teacher directs:*

1. Add it to 29 times itself, using a short method.

2. Subtract it from 26 times itself, using a short method.

3. Multiply it by 125, using a short method.

4. Divide it by 0.125, using a short method.

5. Find $133\frac{1}{3}\%$ of it, using a short method.

This Material for Daily Drill is so arranged as to give daily practice in the fundamental operations. By first going through all the exercises, using in each exercise the number denoted by (a), and then using the number denoted by (b), and so on, more than a hundred different exercises will be found, or more than enough for each recitation in the half year.

EXERCISE 3

Taking (a) *46.96*, (b) *63.82*, (c) *79.78*, (d) *92.54*, (e) *99.46*, *or* (f) *116.92*, *as the teacher directs:*

1. Add it to $3\frac{1}{4} + 12\frac{5}{8} + 17\frac{1}{2} + 6\frac{7}{8} + 4\frac{3}{16} + 12\frac{1}{2} + 3\frac{3}{4}$.
2. Subtract it from twice itself.
3. Multiply it by $12\frac{1}{2}$; by 125; by 1250.
4. Divide it by $3\frac{1}{3}$; by $33\frac{1}{3}$; by $0.33\frac{1}{3}$.
5. Find $16\frac{2}{3}\%$ of it; $1\frac{2}{3}\%$ of it; $0.1\frac{2}{3}\%$ of it.

EXERCISE 4

Taking (a) *0.3524*, (b) *0.5672*, (c) *1.1264*, (d) *1.515*, (e) *1.6552*, *or* (f) *1.9566*, *as the teacher directs:*

1. Add it to $0.296 + 1.128 + 2.78097 + 1.46 + 93.8$.
2. Subtract it from $3.33\frac{1}{3}$.
3. Multiply it by itself.
4. Divide it by $\frac{1}{4}$ of itself; by 4 times itself.
5. Multiply it by $\frac{2}{3} \div \frac{2}{5}$; by $\frac{2}{5} \div \frac{2}{3}$; by $\frac{2}{5}$ of $\frac{2}{3}$.

EXERCISE 5

Taking (a) *2974*, (b) *3378*, (c) *3652*, (d) *5418*, (e) *6016*, *or* (f) *6180*, *as the teacher directs:*

1. Add it to 124 times itself.
2. Subtract it from 100,000; from 27,608.
3. Multiply it by 375; by $37\frac{1}{2}$; by $3\frac{3}{4}$; by $\frac{3}{8}$.
4. Divide it by 625; by $62\frac{1}{2}$; by $6\frac{1}{4}$; by $\frac{5}{8}$.
5. Find $12\frac{1}{2}\%$ of 8 times the number.

EXERCISE 6

Taking (a) *$350.80*, (b) *$536.70*, (c) *$852.12*, (d) *$964.76*, (e) *$1056.56*, *or* (f) *$1073.74*, *as the teacher directs:*

1. Add it to $384.96 + $728.38 + $87.42 + $296.87.
2. Subtract it from $2000.
3. Multiply it by 37.5; by $\frac{3}{8}$; by $3\frac{3}{4}$; by 3750.
4. Divide it by 0.82.
5. Find 6% of it; 5% of it; $5\frac{1}{2}$% of it; $2\frac{1}{2}$% of it.

EXERCISE 7

Taking (a) *4 ft. 6 in.*, (b) *5 ft. 6 in.*, (c) *6 ft. 8 in.*, (d) *7 ft. 4 in.*, (e) *8 ft. 2 in.*, *or* (f) *9 ft. 10 in.*, *as the teacher directs:*

1. Add it to 7 ft. 8 in. + 9 ft. 9 in.
2. Subtract it from 12 ft.
3. Multiply it by $2\frac{1}{2}$; by 25; by $\frac{1}{4}$.
4. Divide it by 2; by 4; by 7; by 12; by $\frac{1}{2}$.
5. Find 75% of it; 50% of it; $66\frac{2}{3}$% of it.

EXERCISE 8

Taking (a) *5 lb.*, (b) *$7\frac{1}{2}$ lb.*, (c) *$4\frac{1}{4}$ lb.*, (d) *$5\frac{3}{4}$ lb.*, (e) *12 oz. or* (f) *3 lb. 4 oz.*, *as the teacher directs:*

1. Add it to 24 oz.
2. Subtract it from $20\frac{1}{2}$ lb.
3. Multiply it by 16; by 48; by 64.
4. Divide it by 3; by 5; by 10; by $\frac{2}{3}$.
5. Find the number of which it is $33\frac{1}{3}$%; $12\frac{1}{2}$%.

EXERCISE 9

Taking (a) $\frac{3}{4}$, (b) $\frac{2}{3}$, (c) $\frac{3}{8}$, (d) $\frac{2}{5}$, (e) $\frac{5}{8}$, *or* (f) $\frac{3}{16}$, *as the teacher directs:*

1. Add it to $0.82 + 3.75 + 2\frac{1}{2} + \frac{8}{5} + 3.53$.
2. Subtract it from 10.17.
3. Multiply it by 3.3.
4. Divide it by $\frac{1}{4}$ of itself.
5. Multiply \$500 by it; 45 ft. by it; 16 lb. 4 oz. by it.

EXERCISE 10

Taking (a) *6%*, (b) *7%*, (c) *$5\frac{1}{2}$%*, (d) *4%*, (e) *$4\frac{1}{2}$%*, *or* (f) *$2\frac{1}{2}$%*, *as the teacher directs:*

1. Add it to 100%; to 25%; to $17\frac{1}{2}$%.
2. Subtract it from 1.
3. Multiply \$14,896.75 by it.
4. Divide 0.19 by it; divide it by 0.19.
5. Divide it by $\frac{2}{3}$ of $\frac{1}{8}$.

EXERCISE 11

Taking (a) *250%*, (b) *137%*, (c) *$233\frac{1}{3}$%*, (d) *125%*, (e) *$166\frac{2}{3}$%*, *or* (f) *$187\frac{1}{2}$%*, *as the teacher directs:*

1. Add it to 3; to 10; to 20.
2. Subtract it from 6; from 10; from 20.
3. Multiply it by 100; by 1000; by 2700.
4. Divide it by 25 times itself.
5. Multiply \$2744.84 by it and add \$182 to the product.

EXERCISE 12

Taking (a) *25.668*, (b) *72.174*, (c) *58.736*, (d) *40.018*, (e) *76.076*, *or* (f) *84.15*, *as the teacher directs:*

1. Add it to 99 times itself.
2. Subtract it from 126 times itself.
3. Multiply it by itself.
4. Divide it by $\frac{1}{3}$; by $33\frac{1}{3}\%$; by $\frac{2}{3}$; by $66\frac{2}{3}\%$.
5. Find 25% of it; 125% of it; $2\frac{1}{2}\%$ of it; 0.25% of it.

EXERCISE 13

Taking (a) *269.74*, (b) *473.84*, (c) *856.78*, (d) *299.52*, (e) *1240.18*, *or* (f) *1404.76*, *as the teacher directs:*

1. Add it to $2.73 + 85 + 0.91 + 388 + 148.36 + 298.37$.
2. Subtract it from 2063.009; from 2000.
3. Multiply it by itself.
4. Divide it by 3.82; by 1.91; by 0.955.
5. Find $16\frac{2}{3}\%$ of it; $8\frac{1}{3}\%$ of it; $4\frac{1}{6}\%$ of it.

EXERCISE 14

Taking (a) *35.6 m.*, (b) *58.6 cm.*, (c) *297.8 mm.*, (d) *1.5 cm.*, (e) *0.156 m.*, *or* (f) *53.96 cm.*, *as the teacher directs:*

1. Add it to 72.008 m. + 2.96 m. + 48.072 m.
2. Subtract it from 1000 m.
3. Multiply it by $\frac{1}{2}$; by 50%; by 0.5.
4. Divide it by $\frac{3}{4}$; by 0.75; by 75%.
5. Find 300% of it; $333\frac{1}{3}\%$ of it; $33\frac{1}{3}\%$ of it; $3\frac{1}{3}\%$ of it.

EXERCISE 15

Taking (a) *62 yd. 8 in.*, (b) *26 yd. 9 in.*, (c) *32 yd. 26 in.*, (d) *5 yd. 18 in.*, (e) *8 yd. 31 in.*, *or* (f) *9 yd. 17 in.*, *as the teacher directs:*

1. Add it to 6 yd. 18 in. + 9 yd. 30 in. + 4 yd. 3 in.
2. Subtract it from 90 yd.
3. Multiply it by 2; by 3; by $2\frac{1}{2}$; by 3.1.
4. Divide it by $\frac{1}{2}$; by $\frac{1}{3}$; by $\frac{2}{3}$; by $\frac{1}{4}$; by $\frac{3}{4}$.
5. Find $\frac{3}{4}$ of it; $\frac{3}{8}$ of it; $\frac{2}{3}$ of it.

EXERCISE 16

Taking (a) *2973.8*, (b) *5476.8*, (c) *299.64*, (d) *1256.74*, (e) *1018.18*, *or* (f) *24.876*, *as the teacher directs:*

1. Add it to 27.857 + 938.096 + 2.98 + 47 + 42.68.
2. Subtract it from 8028.003.
3. Multiply it by 700.2.
4. Divide it by 700.2; by 350.1; by 175.05.
5. Find $\frac{3}{4}$ of it; divide it by $\frac{3}{4}$; multiply it by $1\frac{1}{3}$.

EXERCISE 17

Taking (a) *$556.76*, (b) *$1025.74*, (c) *$19,746.80*, (d) *$2551*, (e) *$653.68*, *or* (f) *$5873.70*, *as the teacher directs:*

1. Add it to $32\frac{1}{3}$ times itself.
2. Subtract it from $67\frac{2}{3}$ times itself.
3. Multiply it by 207.4.
4. Divide it by 37.8.
5. Find 6% of it; 106% of it; 10.6% of it.

TABLES FOR REFERENCE

Length

12 inches (in.)	= 1 foot (ft.)
3 feet	= 1 yard (yd.)
$5\frac{1}{2}$ yards, or $16\frac{1}{2}$ feet	= 1 rod (rd.)
320 rods, or 5280 feet	= 1 mile (mi.)

Square Measure

144 square inches (sq. in.)	= 1 square foot (sq. ft.)
9 square feet	= 1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards	= 1 square rod (sq. rd.)
160 square rods	= 1 acre (A.)
640 acres	= 1 square mile (sq. mi.)

Cubic Measure

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27 cubic feet	= 1 cubic yard (cu. yd.)
128 cubic feet	= 1 cord (cd.)

Weight

16 ounces (oz.)	= 1 pound (lb.)
2000 pounds	= 1 ton (T.)

Liquid Measure

4 gills (gi.)	= 1 pint (pt.)
2 pints	= 1 quart (qt.)
4 quarts	= 1 gallon (gal.)
$31\frac{1}{2}$ gallons	= 1 barrel (bbl.)
2 barrels	= 1 hogshead (hhd.)

Dry Measure

2 pints (pt.) = 1 quart (qt.)
8 quarts = 1 peck (pk.)
4 pecks = 1 bushel (bu.)

Time

60 seconds (sec.) = 1 minute (min.)
60 minutes = 1 hour (hr.)
24 hours = 1 day (da.)
7 days = 1 week (wk.)
12 months (mo.) = 1 year (yr.)
365 days = 1 common year
366 days = 1 leap year

Value

10 mills = 1 cent (¢ or ct.)
10 cents = 1 dime (d.)
10 dimes = 1 dollar ($)

Angles and Arcs

60 seconds ($60''$) = 1 minute ($1'$)
60 minutes = 1 degree (1°)

Surveyors' Table of Length

7.92 inches = 1 link (li.)
100 links = 4 rods = 1 chain (ch.)
80 chains = 5280 feet = 1 mile

Counting

12 units = 1 dozen (doz.)
12 dozen, or 144 units = 1 gross (gro.)

Paper

24 sheets = 1 quire (qr.)
500 sheets = 1 ream

English Money

4 farthings = 1 penny (d.)
12 pence = 1 shilling (s. or /)
20 shillings = 1 pound (£) or sovereign

A penny (plural, *pence*) is about 2¢ of our money, a shilling is about 24¢, and a pound is equivalent to $4.866.

German Money

100 pfennigs = 1 mark (M.)

A mark is about 24¢ of our money.

French Money

100 centimes = 1 franc (fr.)

A franc is about 19.3¢ of our money.

Metric Length

10 millimeters (mm.) = 1 centimeter (cm.)
10 centimeters = 1 decimeter (dm.)
10 decimeters = 1 meter (m.)
1000 meters = 1 kilometer (km.)

Metric Weight

1000 grams (g.) = 1 kilogram (kg.)
1000 kilograms = 1 metric ton (t.)

Metric Capacity

100 liters (l.) = 1 hektoliter (hl.)

Practical Equivalents

231 cu. in. = 1 gal.	$1\frac{1}{4}$ cu. ft. = 1 bu.
$31\frac{1}{2}$ gal. = 1 bbl.	$7\frac{1}{2}$ gal. = 1 cu. ft.

More accurately speaking, 1 bu. contains 2150.42 cu. in. and 1 cu. ft. contains 7.48 gal. Barrels vary in size, but a barrel is often considered to be equivalent to $31\frac{1}{2}$ gal. A perch of masonry varies, but it is generally taken as about 25 cu. ft., or, more exactly, $24\frac{3}{4}$ cu. ft.

Powers and Roots

No.	Squares	Cubes	Square Roots	Cube Roots	No.	Squares	Cubes	Square Roots	Cube Roots
1	1	1	1.000	1.000	51	2,601	132,651	7.141	3.708
2	4	8	1.414	1.260	52	2,704	140,608	7.211	3.733
3	9	27	1.732	1.442	53	2,809	148,877	7.280	3.756
4	16	64	2.000	1.587	54	2,916	157,464	7.348	3.780
5	25	125	2.236	1.710	55	3,025	166,375	7.416	3.803
6	36	216	2.449	1.817	56	3,136	175,616	7.483	3.826
7	49	343	2.646	1.913	57	3,249	185,193	7.550	3.849
8	64	512	2.828	2.000	58	3,364	195,112	7.616	3.871
9	81	729	3.000	2.080	59	3,481	205,379	7.681	3.893
10	100	1,000	3.162	2.154	60	3,600	216,000	7.746	3.915
11	121	1,331	3.317	2.224	61	3,721	226,981	7.810	3.936
12	144	1,728	3.464	2.289	62	3,844	238,328	7.874	3.958
13	169	2,197	3.606	2.351	63	3,969	250,047	7.937	3.979
14	196	2,744	3.742	2.410	64	4,096	262,144	8.000	4.000
15	225	3,375	3.873	2.466	65	4,225	274,625	8.062	4.021
16	256	4,096	4.000	2.520	66	4,356	287,496	8.124	4.041
17	289	4,913	4.123	2.571	67	4,489	300,763	8.185	4.062
18	324	5,832	4.243	2.621	68	4,624	314,432	8.246	4.082
19	361	6,859	4.359	2.668	69	4,761	328,509	8.307	4.102
20	400	8,000	4.472	2.714	70	4,900	343,000	8.367	4.121
21	441	9,261	4.583	2.759	71	5,041	357,911	8.426	4.141
22	484	10,648	4.690	2.802	72	5,184	373,248	8.485	4.160
23	529	12,167	4.796	2.844	73	5,329	389,017	8.544	4.179
24	576	13,824	4.899	2.884	74	5,476	405,224	8.602	4.198
25	625	15,625	5.000	2.924	75	5,625	421,875	8.660	4.217
26	676	17,576	5.099	2.962	76	5,776	438,976	8.718	4.236
27	729	19,683	5.196	3.000	77	5,929	456,533	8.775	4.254
28	784	21,952	5.292	3.037	78	6,084	474,552	8.832	4.273
29	841	24,389	5.385	3.072	79	6,241	493,039	8.888	4.291
30	900	27,000	5.477	3.107	80	6,400	512,000	8.944	4.309
31	961	29,791	5.568	3.141	81	6,561	531,441	9.000	4.327
32	1 024	32,768	5.657	3.175	82	6,724	551,368	9.055	4.344
33	1 089	35,937	5.745	3.208	83	6,889	571,787	9.110	4.362
34	1 156	39,304	5.831	3.240	84	7,056	592,704	9.165	4.380
35	1 225	42,875	5.916	3.271	85	7,225	614,125	9.220	4.397
36	1 296	46,656	6.000	3.302	86	7,396	636,056	9.274	4.414
37	1 369	50,653	6.083	3.332	87	7,569	658,503	9.327	4.431
38	1 444	54,872	6.164	3.362	88	7,744	681,472	9.381	4.448
39	1 521	59,319	6.245	3.391	89	7,921	704,969	9.434	4.465
40	1 600	64,000	6.325	3.420	90	8,100	729,000	9.487	4.481
41	1 681	68,921	6.403	3.448	91	8,281	753,571	9.539	4.498
42	1 764	74,088	6.481	3.476	92	8,464	778,688	9.592	4.514
43	1 849	79,507	6.557	3.503	93	8,649	804,357	9.644	4.531
44	1 936	85,184	6.633	3.530	94	8,836	830,584	9.695	4.547
45	2 025	91,125	6.708	3.557	95	9,025	857,375	9.747	4.563
46	2 116	97,336	6.782	3.583	96	9,216	884,736	9.798	4.579
47	2 209	103,823	6.856	3.609	97	9,409	912,673	9.849	4.595
48	2 304	110,592	6.928	3.634	98	9,604	941,192	9.899	4.610
49	2 401	117,649	7.000	3.659	99	9,801	970,299	9.950	4.626
50	2 500	125,000	7.071	3.684	100	10,000	1,000,000	10.000	4.641

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